



THE PREMIER CONFERENCE & EXHIBITION ON
COMPUTER GRAPHICS & INTERACTIVE TECHNIQUES



SIGGRAPH 2023
LOS ANGELES+ 6-10 AUG

LOCALLY MESHABLE FRAME FIELDS

HENG LIU - UNIVERSITY OF BERN
DAVID BOMMES - UNIVERSITY OF BERN

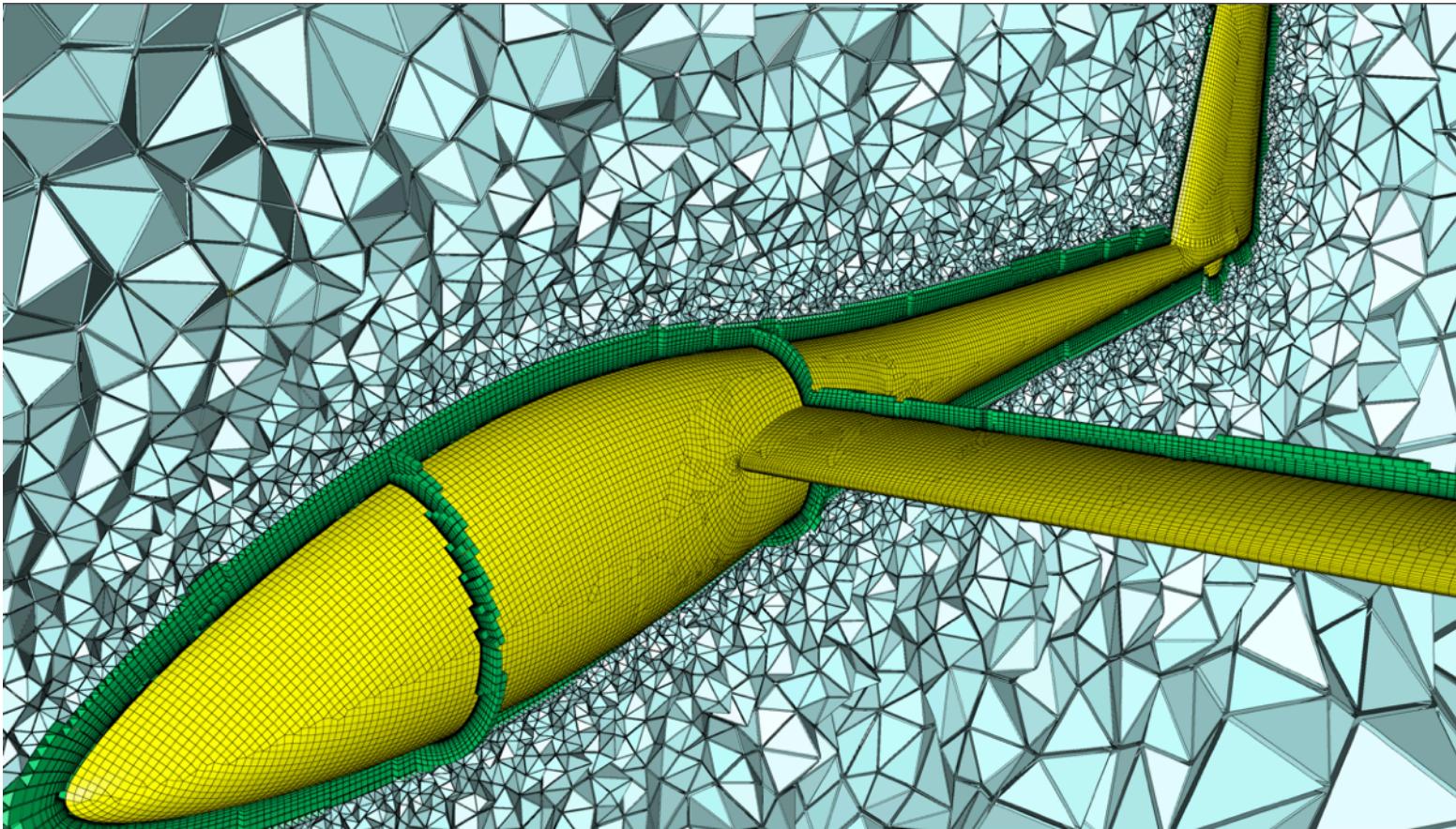
u^b

b
UNIVERSITÄT
BERN

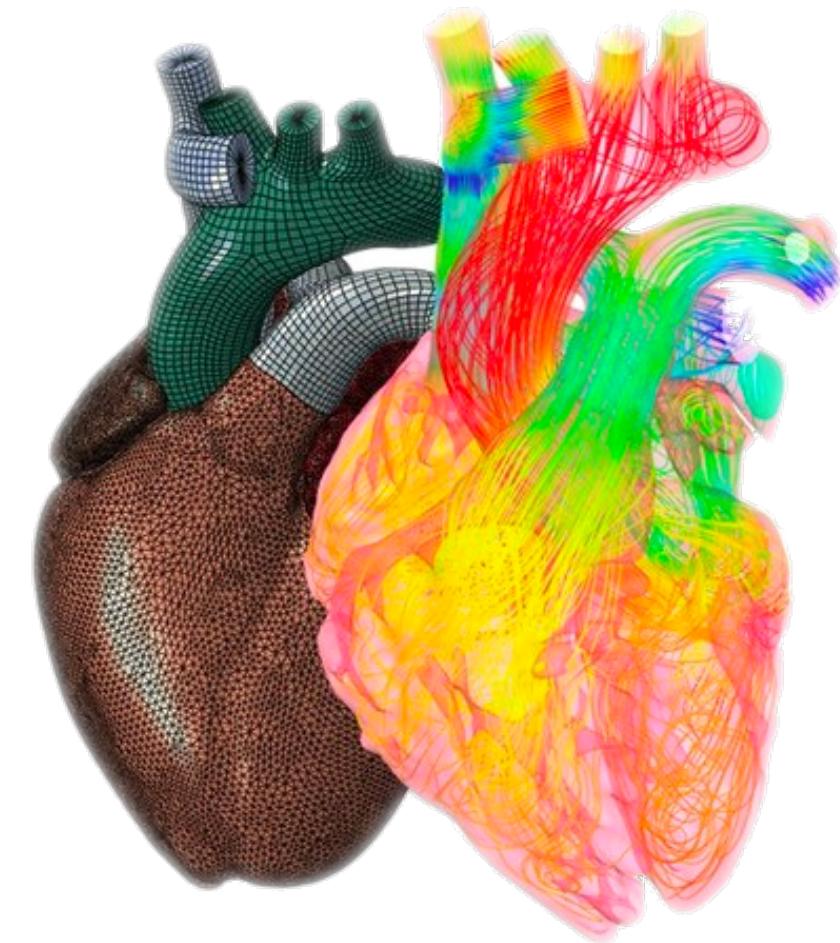




CONTEXT



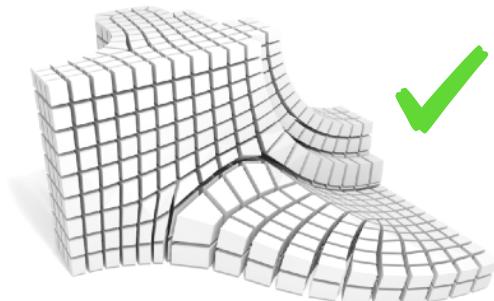
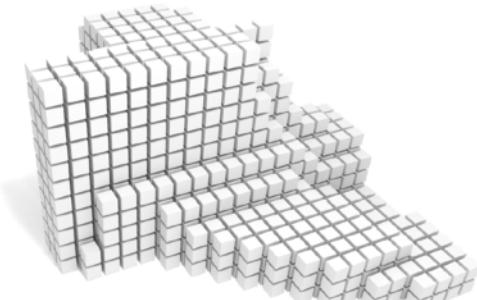
[© Cadence]



[© FlowVision]

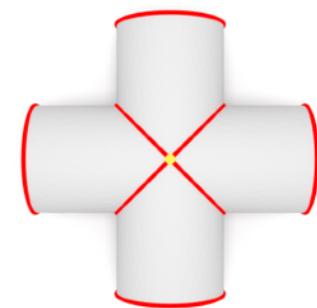
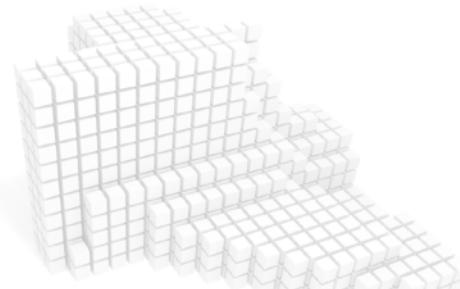
→ HEX MESH QUALITY

- Geometric Fidelity

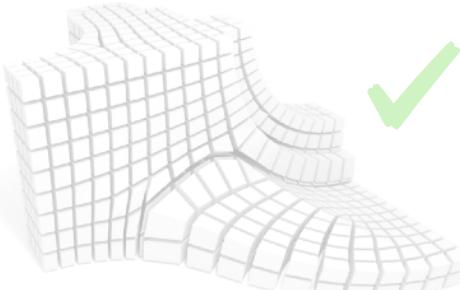


→ HEX MESH QUALITY

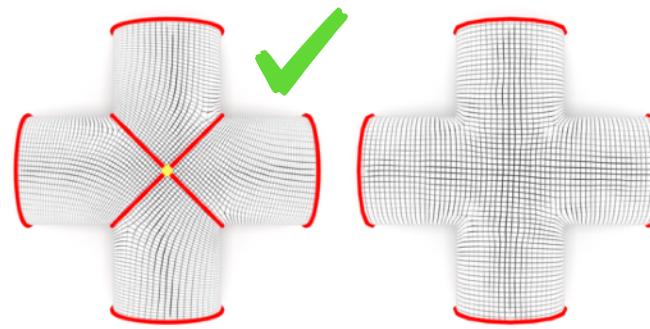
- Geometric Fidelity
- Feature preservation



Input shape



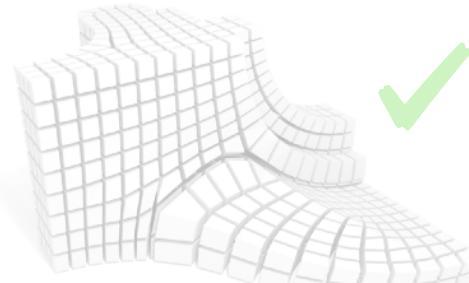
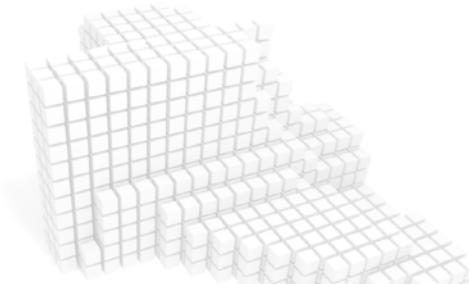
Output hex mesh 1



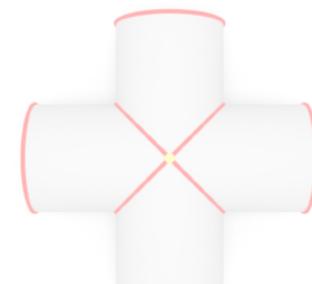
Output hex mesh 2

→ HEX MESH QUALITY

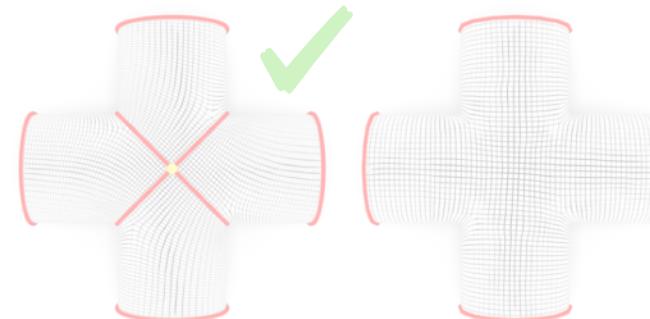
- Geometric Fidelity



- Feature preservation



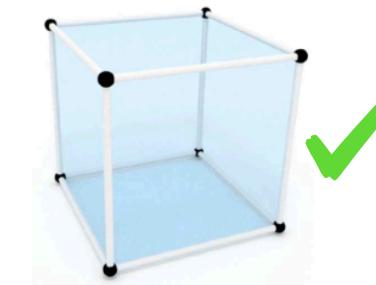
Input shape



Output hex mesh 1

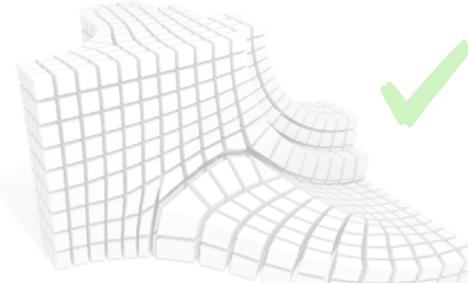
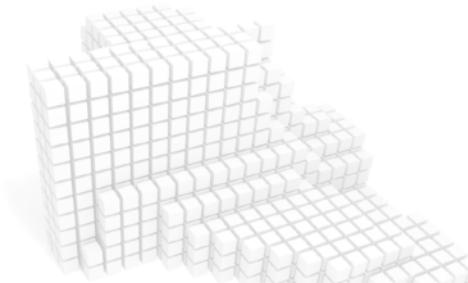
Output hex mesh 2

- Element quality

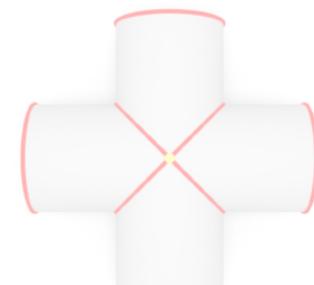


→ HEX MESH QUALITY

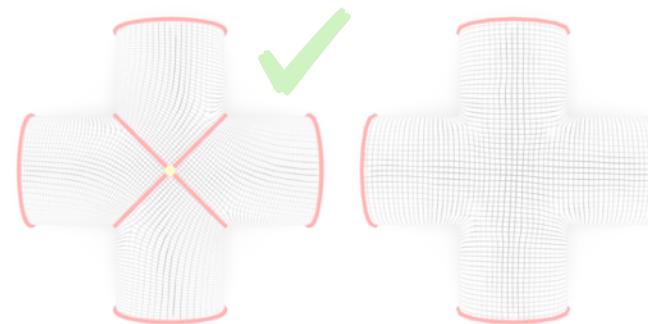
- Geometric Fidelity



- Feature preservation



Input shape



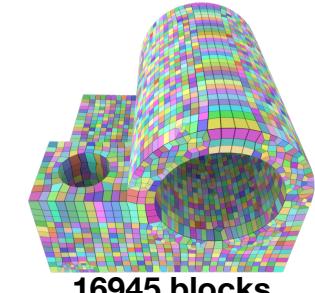
Output hex mesh 1

Output hex mesh 2

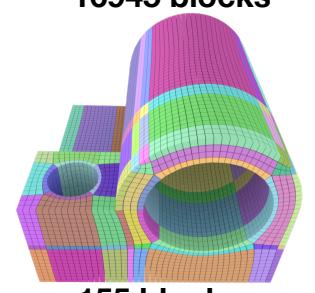
- Element quality



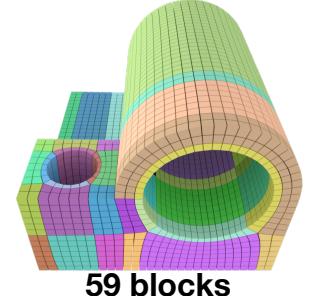
- Regularity



16945 blocks



155 blocks



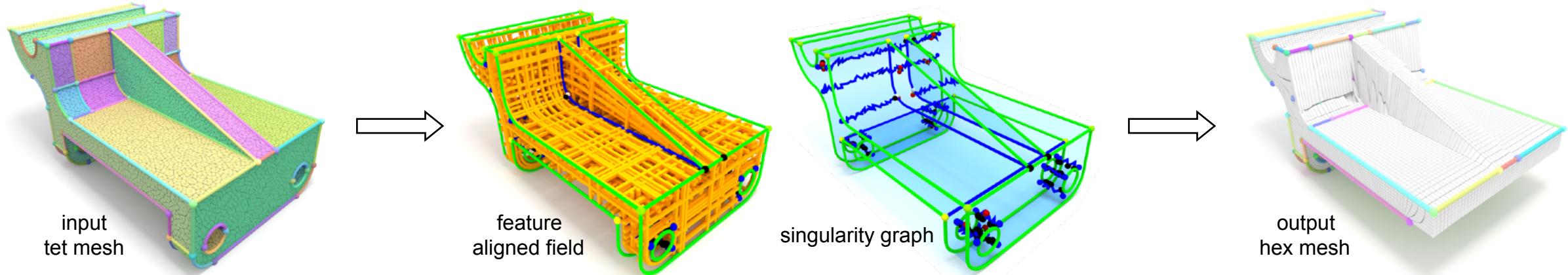
59 blocks



FRAME FIELD BASED HEX MESHING

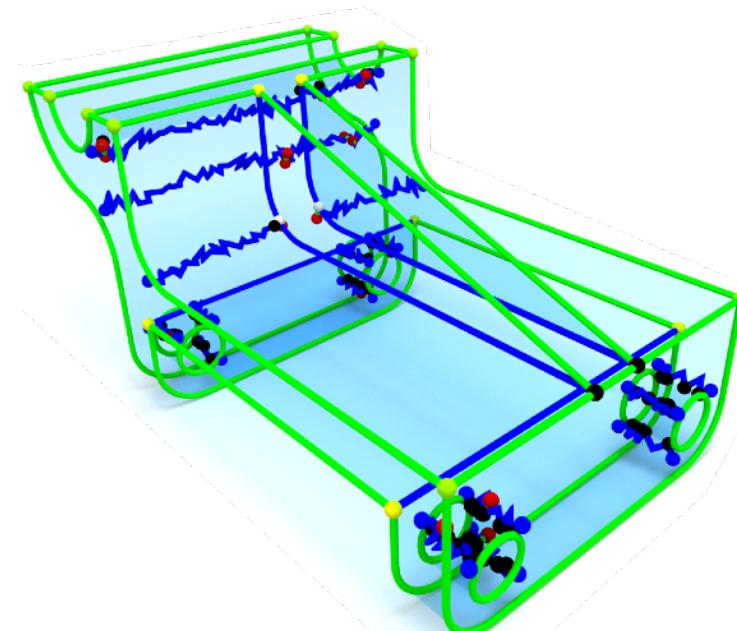
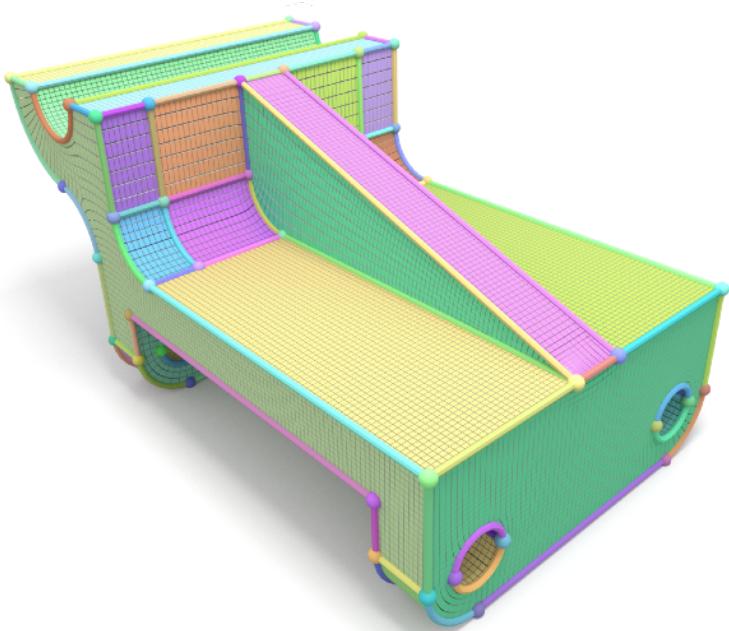


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PROBLEM



**Locally Meshable
Frame Fields**

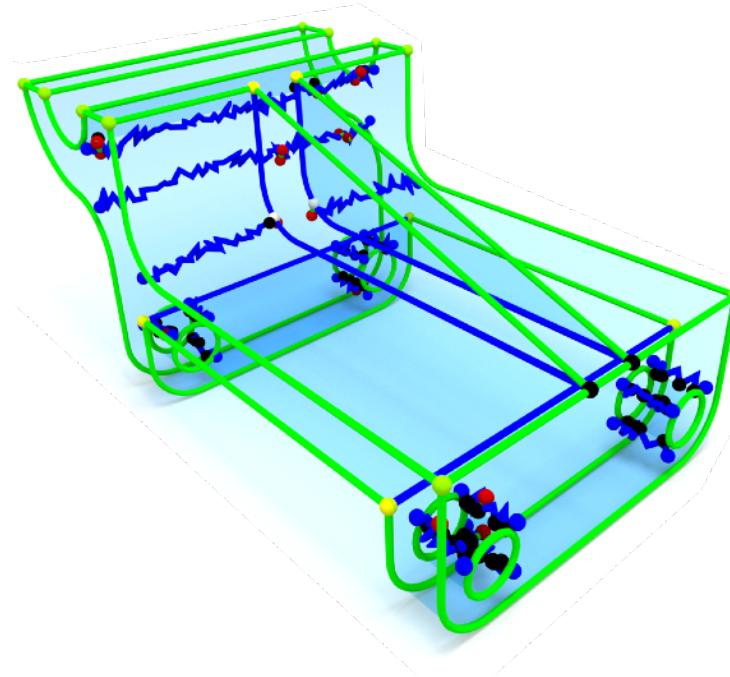
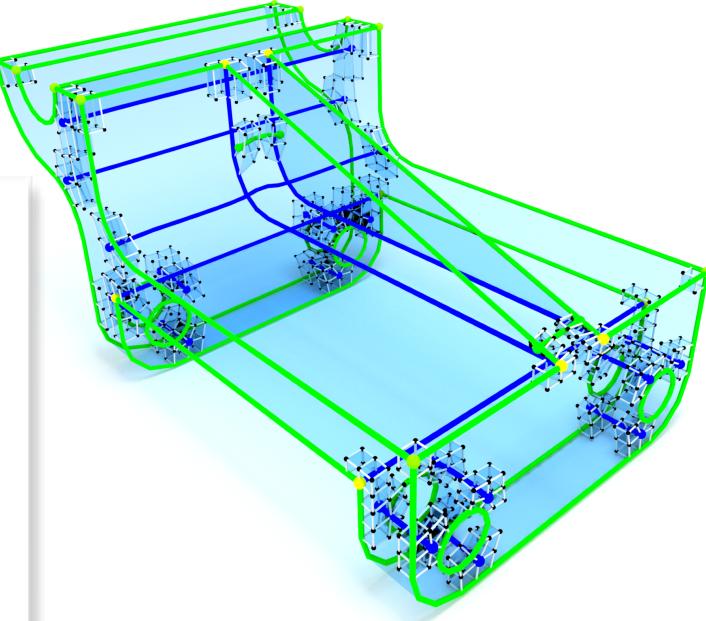
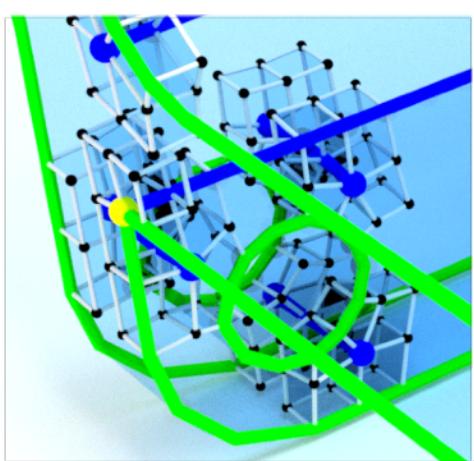
Frame Fields



GOAL



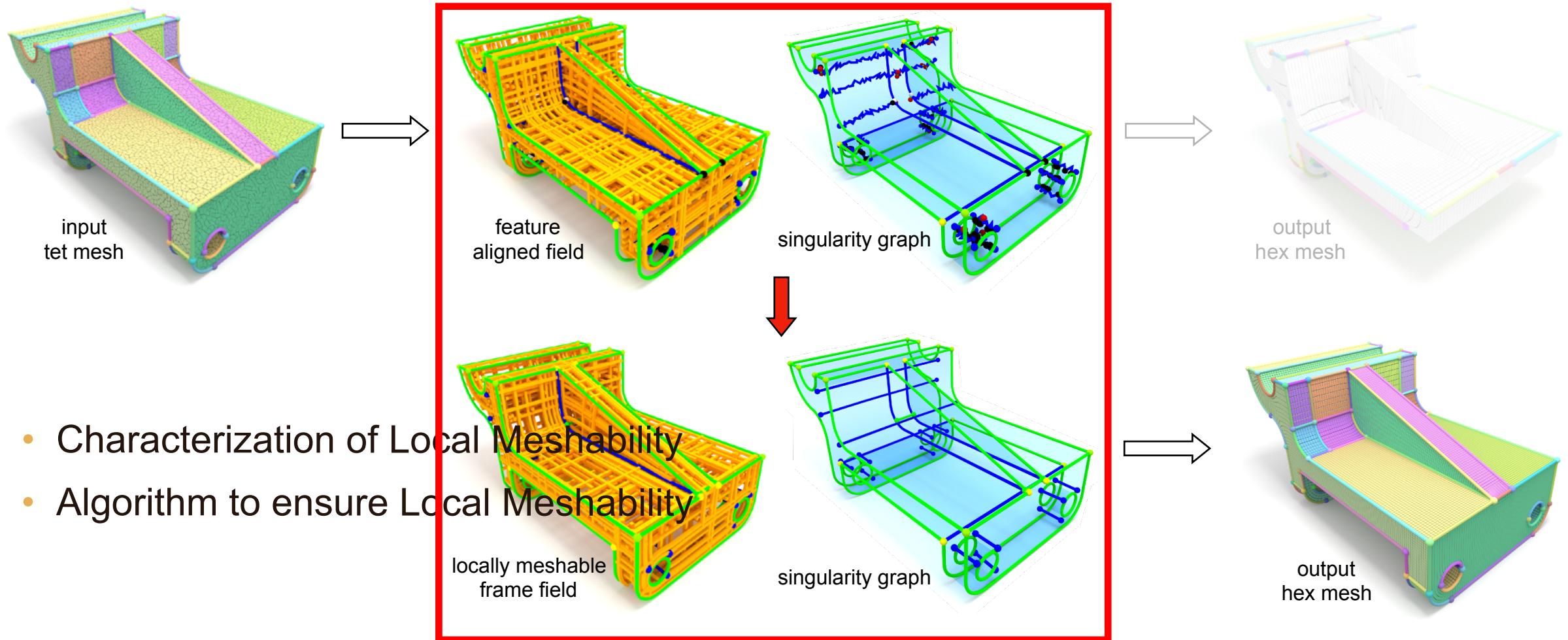
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**Locally Meshable
Frame Fields**

Frame Fields

→ CONTRIBUTIONS



Frame Field Meshability

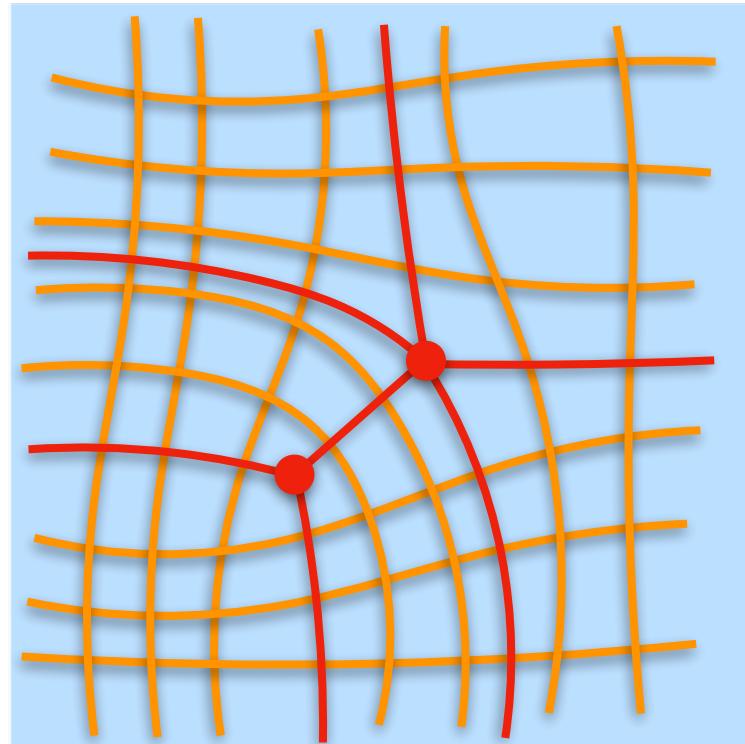
2D



QUAD MESH VS. FRAME FIELD TOPOLOGY

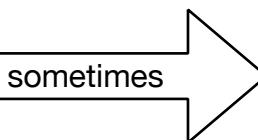
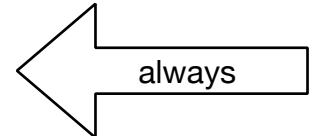


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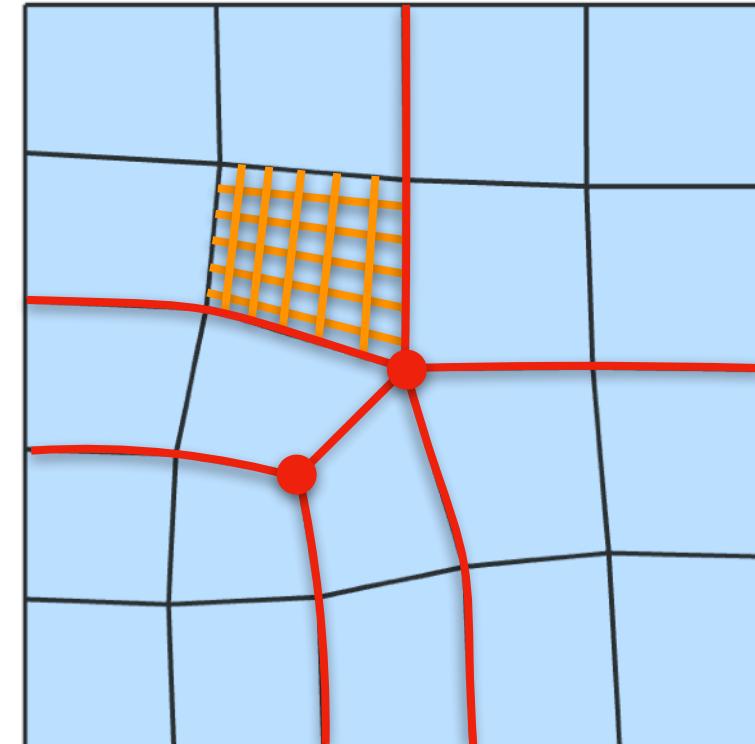
Frame Field

Induced Field



sometimes

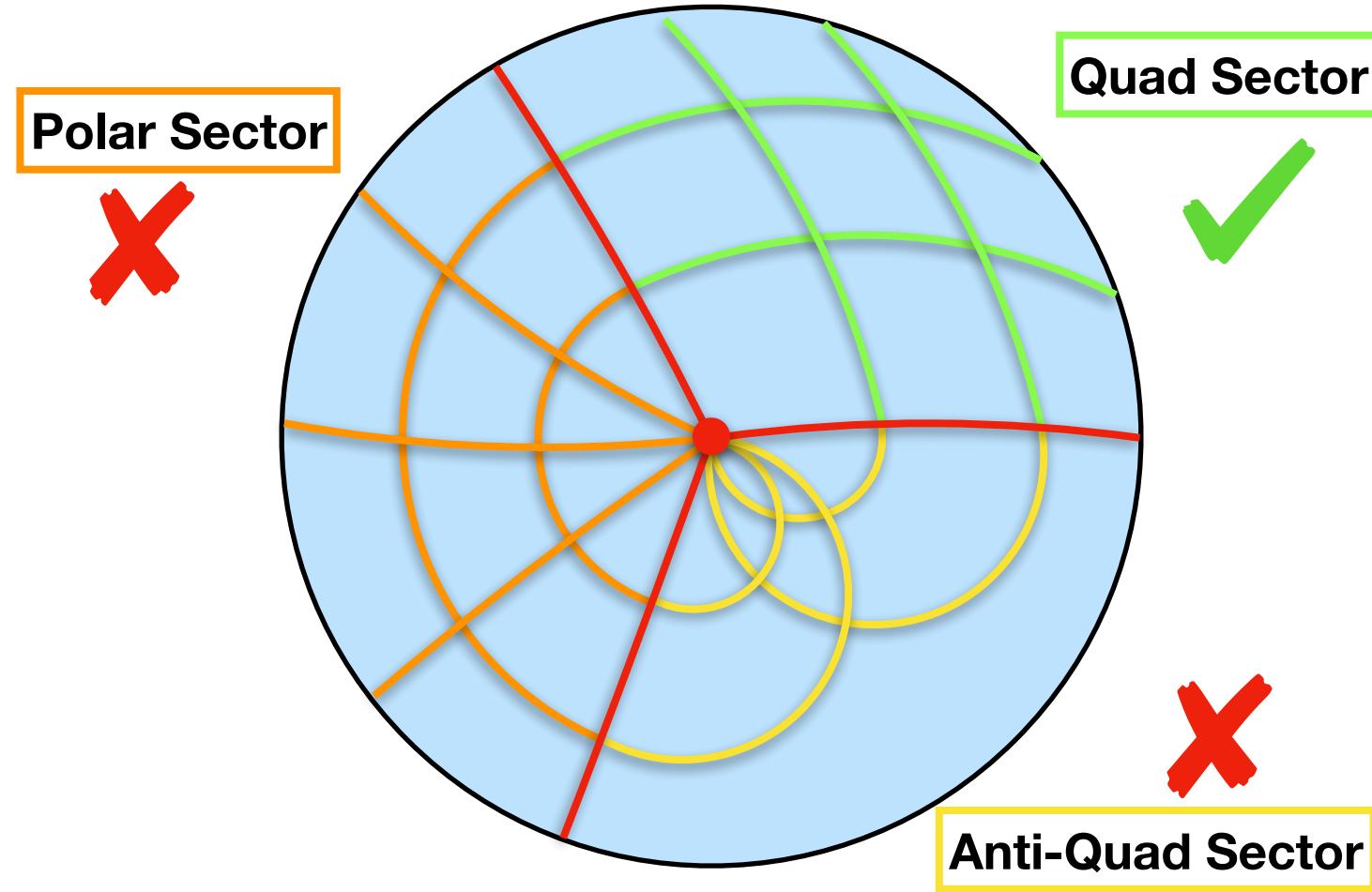
Meshability



Quad Mesh

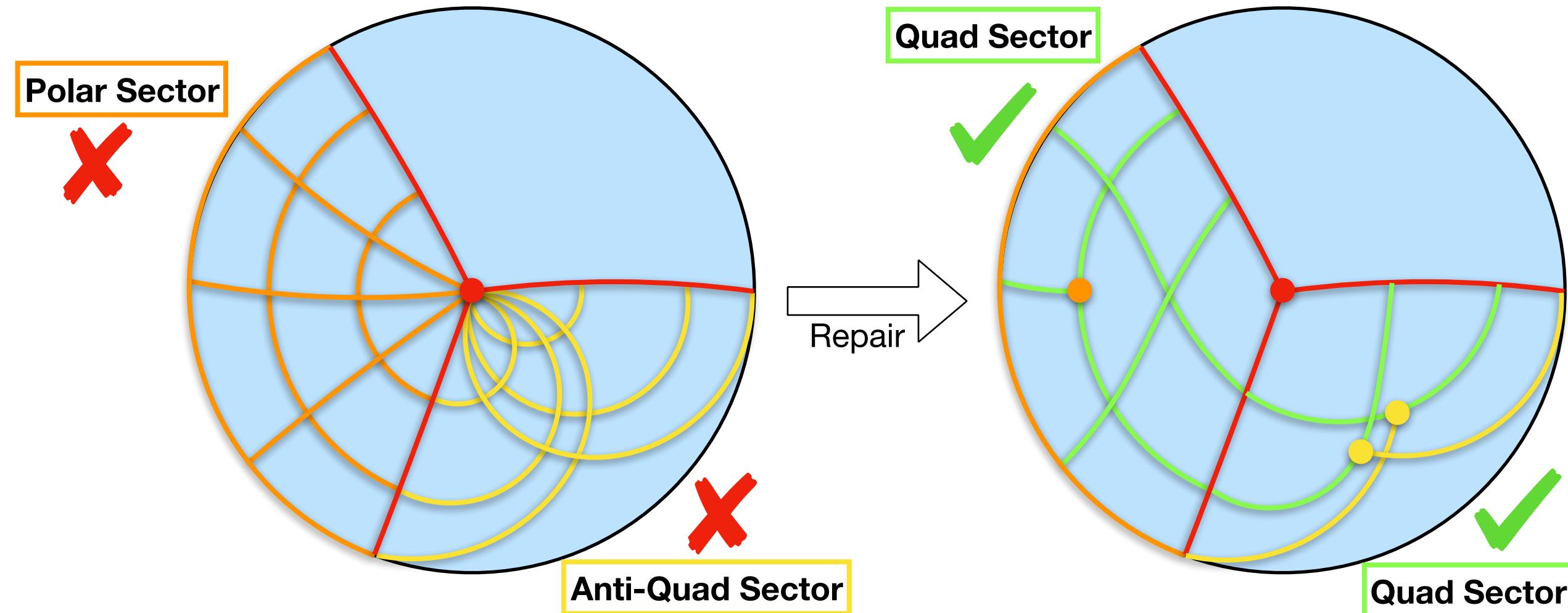


LOCAL MESHABILITY CONDITION





LOCAL MESHABILITY REPAIR

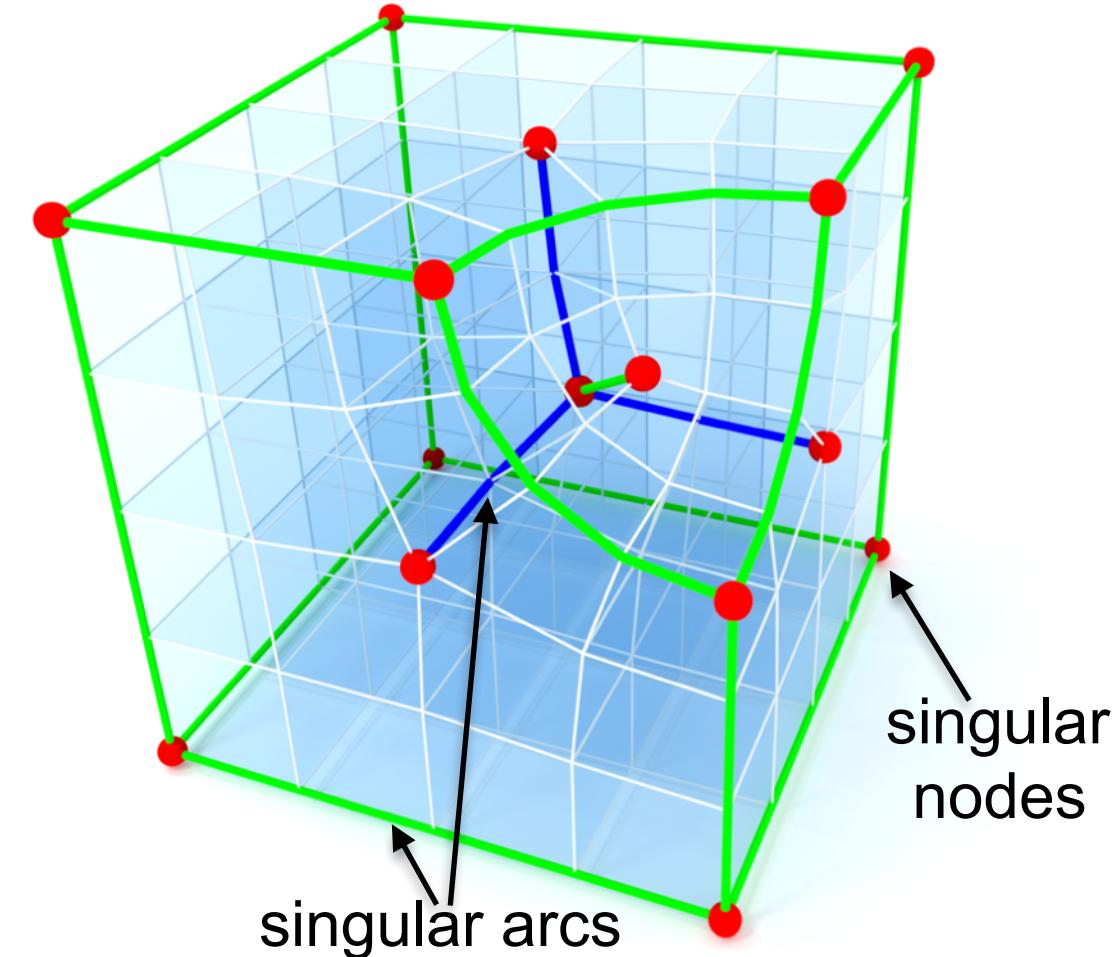
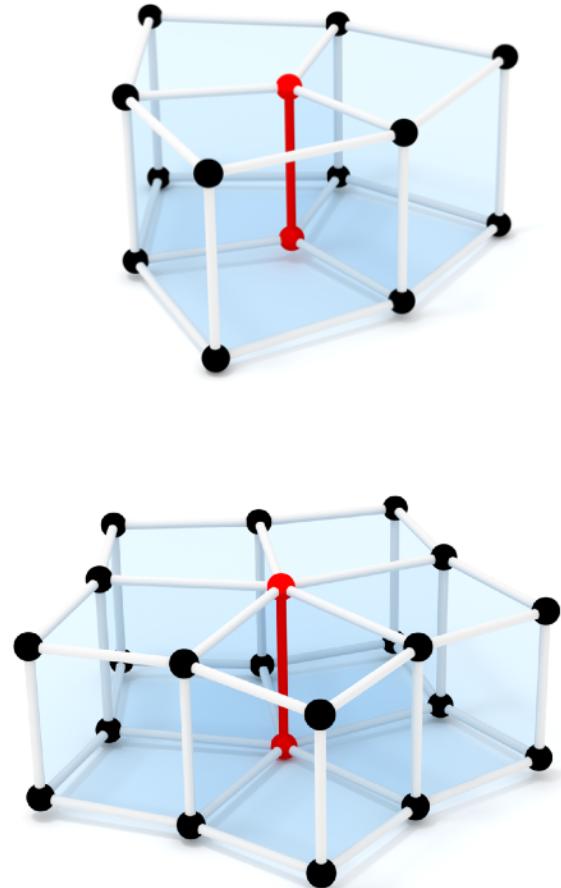


Frame Field Meshability

3D



HEX MESH SINGULAR GRAPH

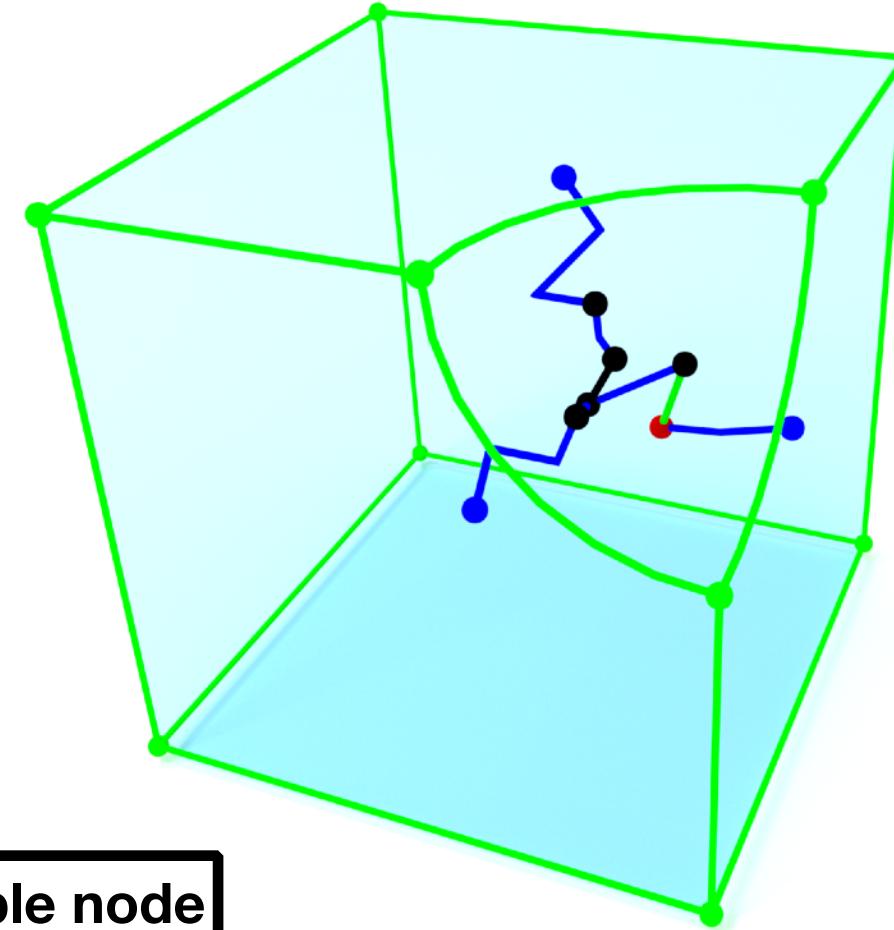
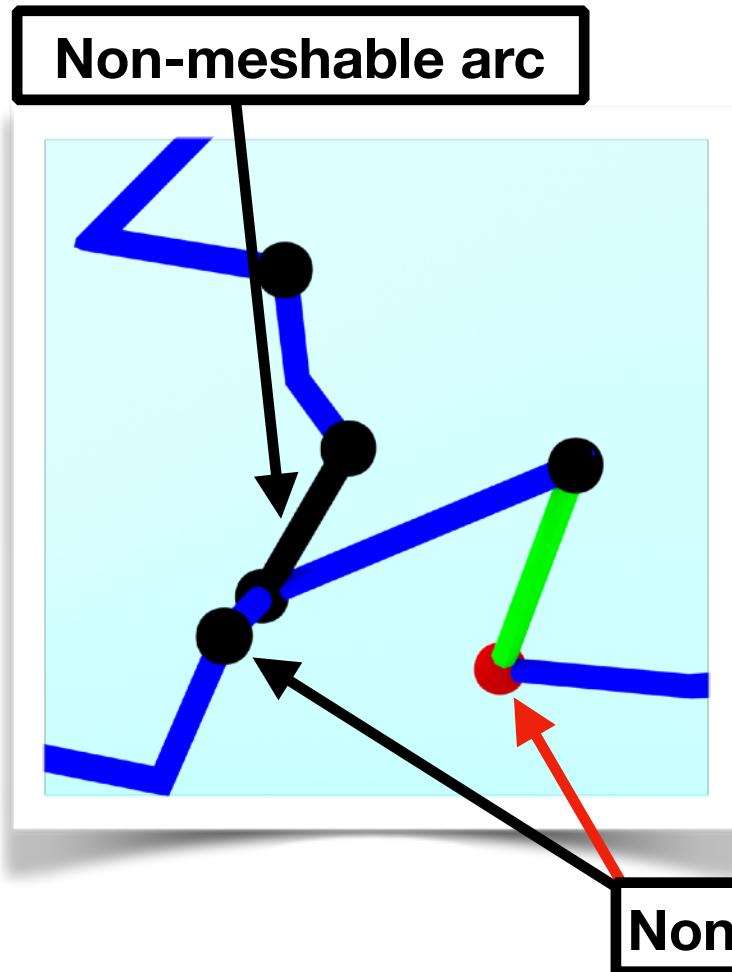




FRAME FIELD SINGULARITIES



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FRAME FIELD SINGULARITIES



Algorithm

1. Repair non-meshable **arcs**
2. Repair non-meshable **nodes**



Arc Meshability

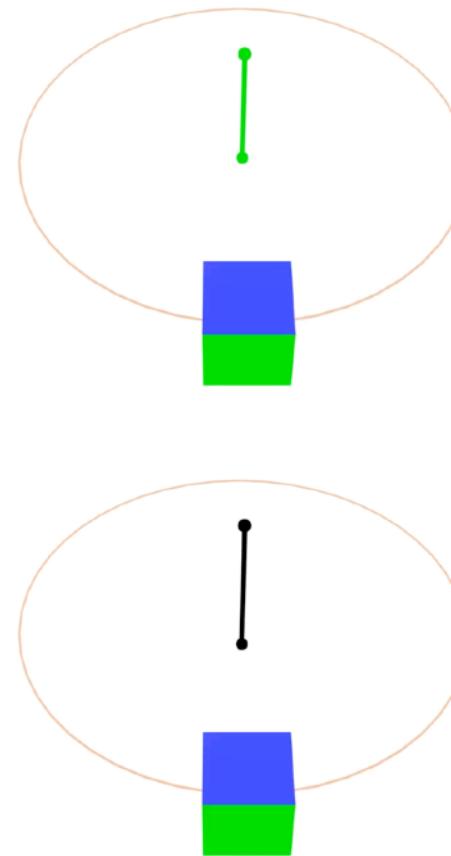
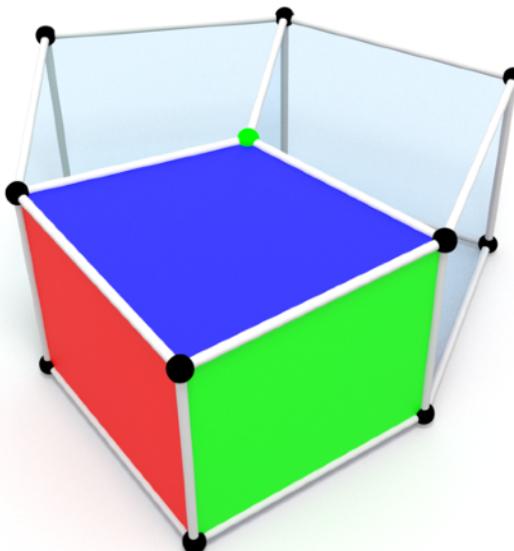


ARC MESHABILITY



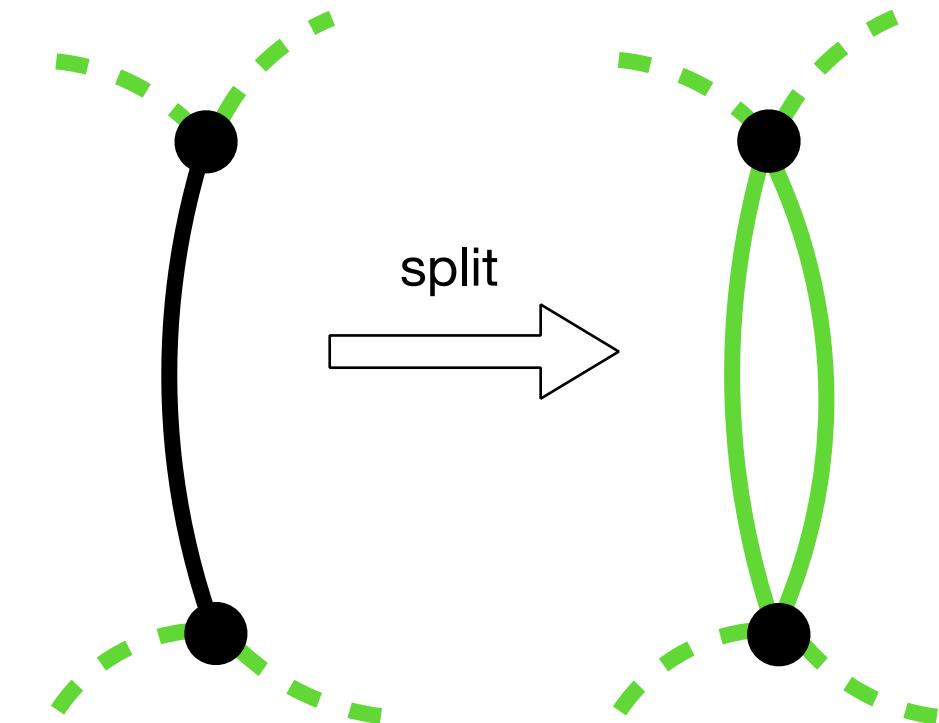
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- Non-meshable Monodromy



- Repair

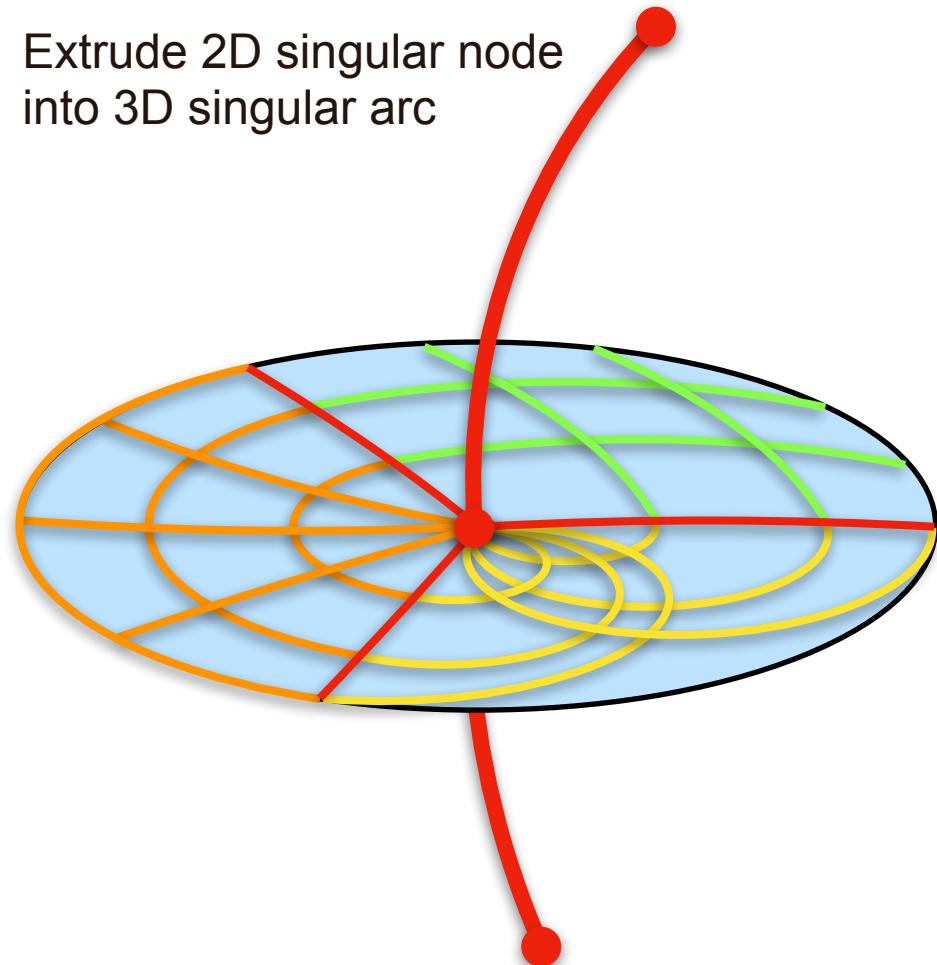
- always splittable into two flow-aligned arcs [Jiang et al. 2014]





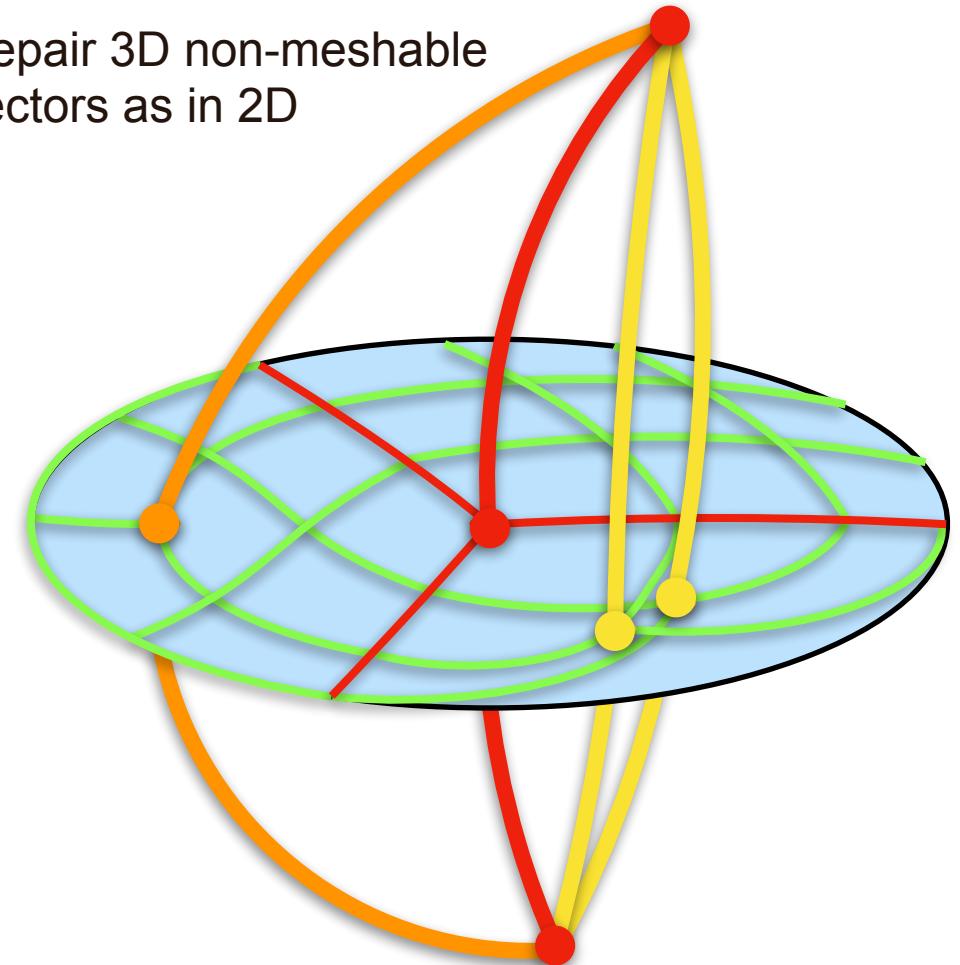
FLOW-ALIGNED SINGULAR ARCS

- Extrude 2D singular node into 3D singular arc



Repair

- Repair 3D non-meshable sectors as in 2D



Node Meshability

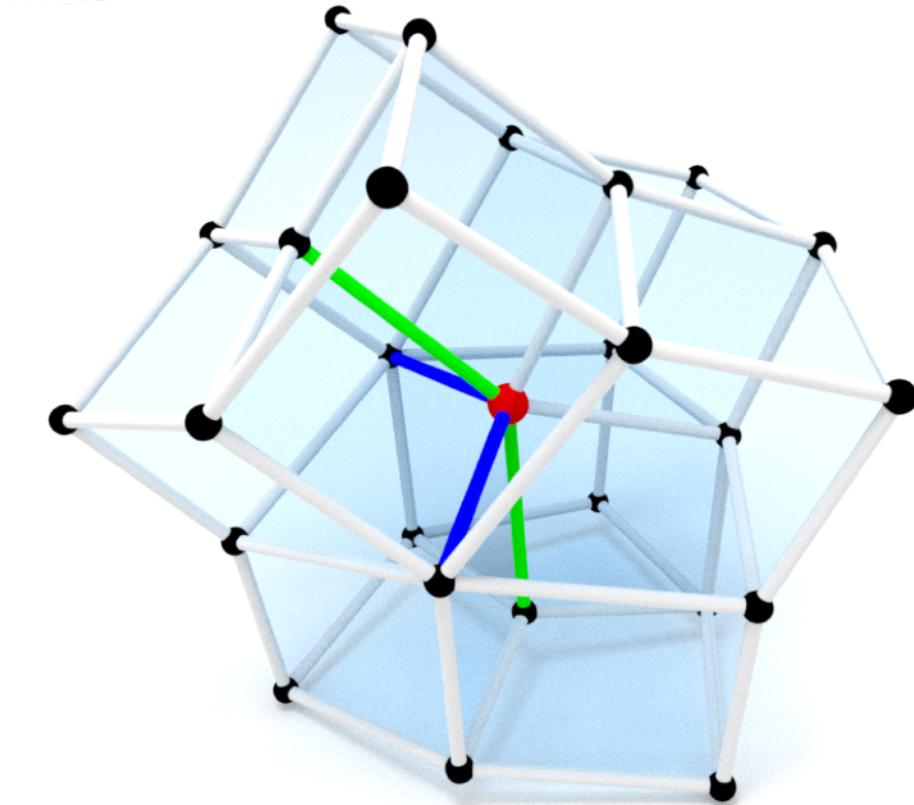
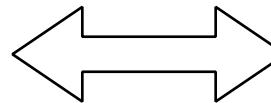
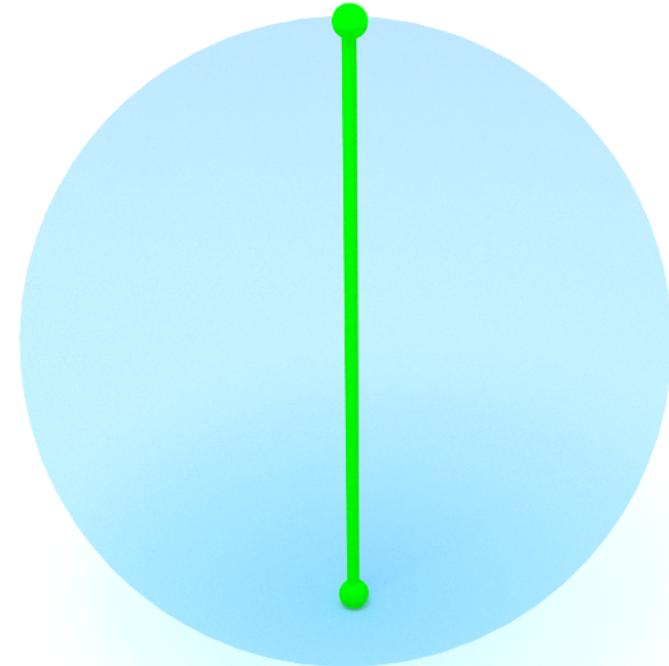


SINGULAR NODES



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- Singular nodes result from the interaction between multiple singular arcs



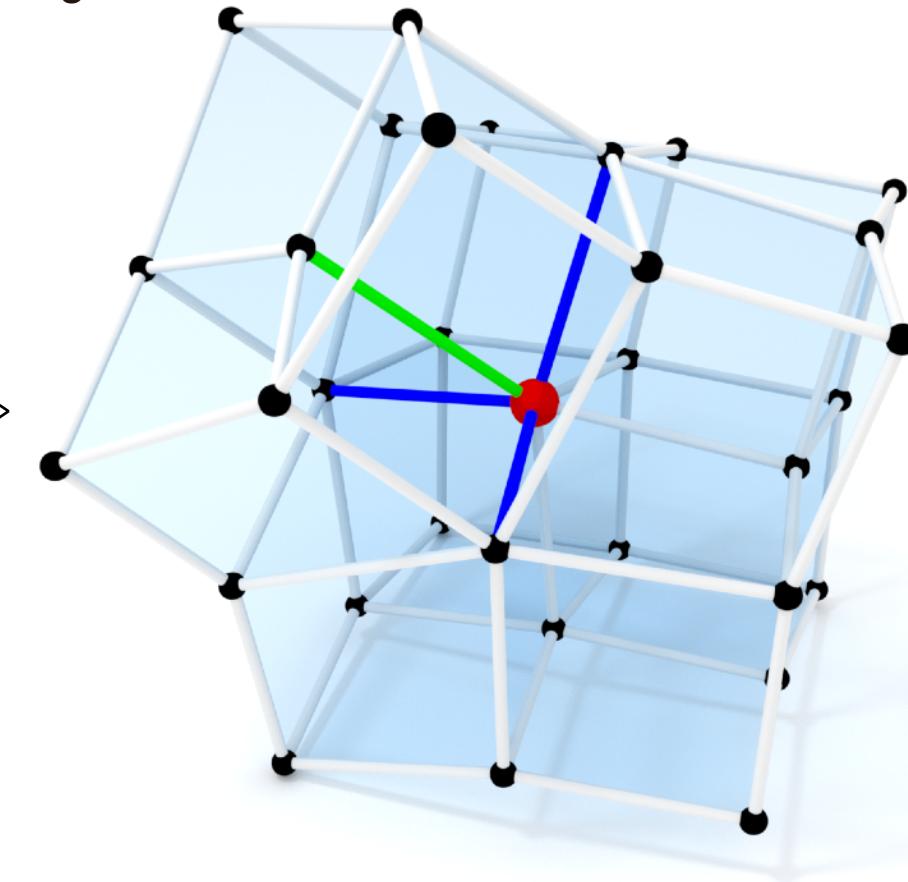
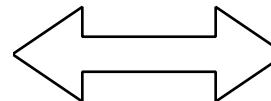
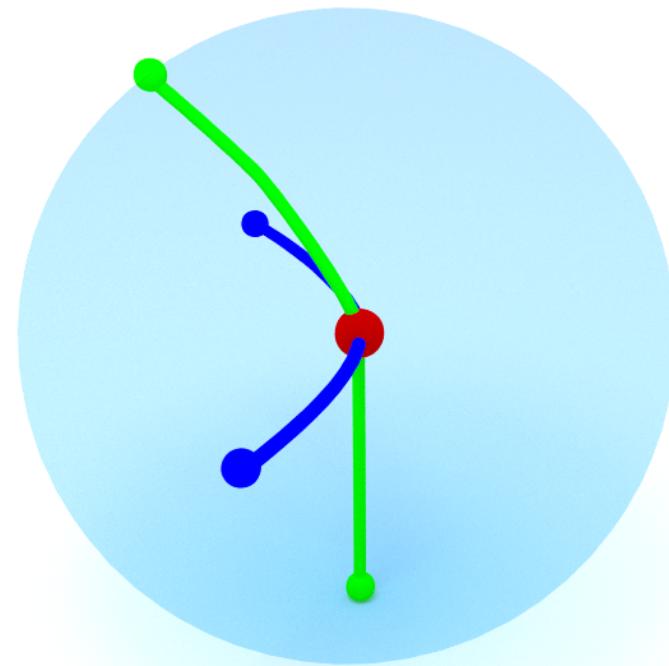


SINGULAR NODES



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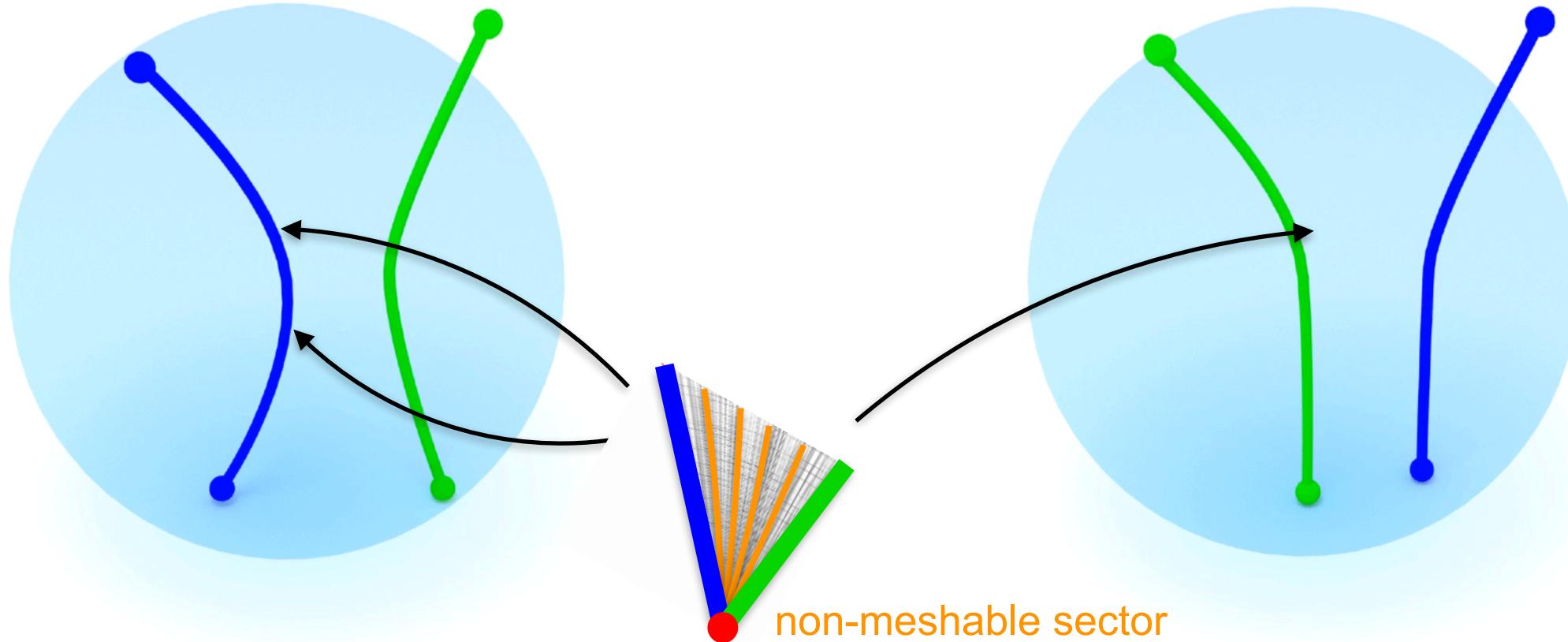
- Singular nodes result from the interaction between multiple singular arcs





NON-MESHABLE INTERACTIONS

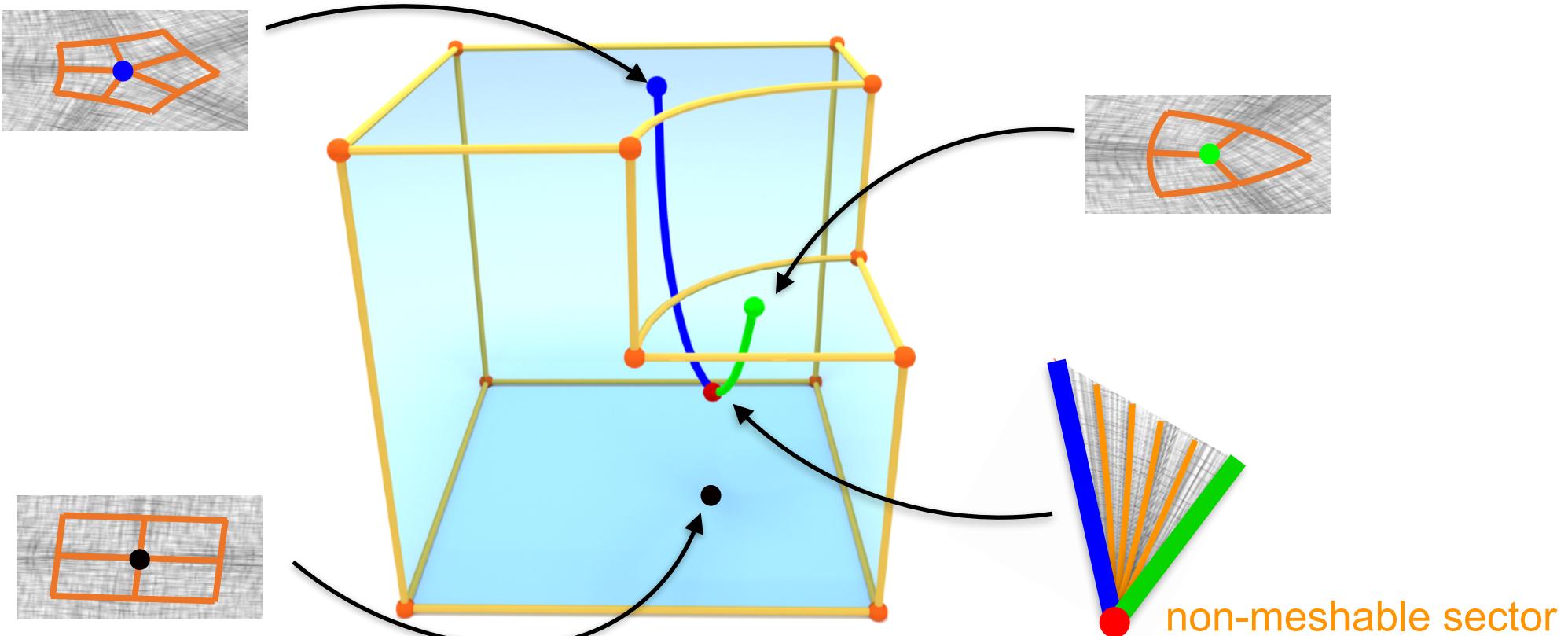
- Partial overlap of parallel arcs





REPAIRING ZIPPER NODES

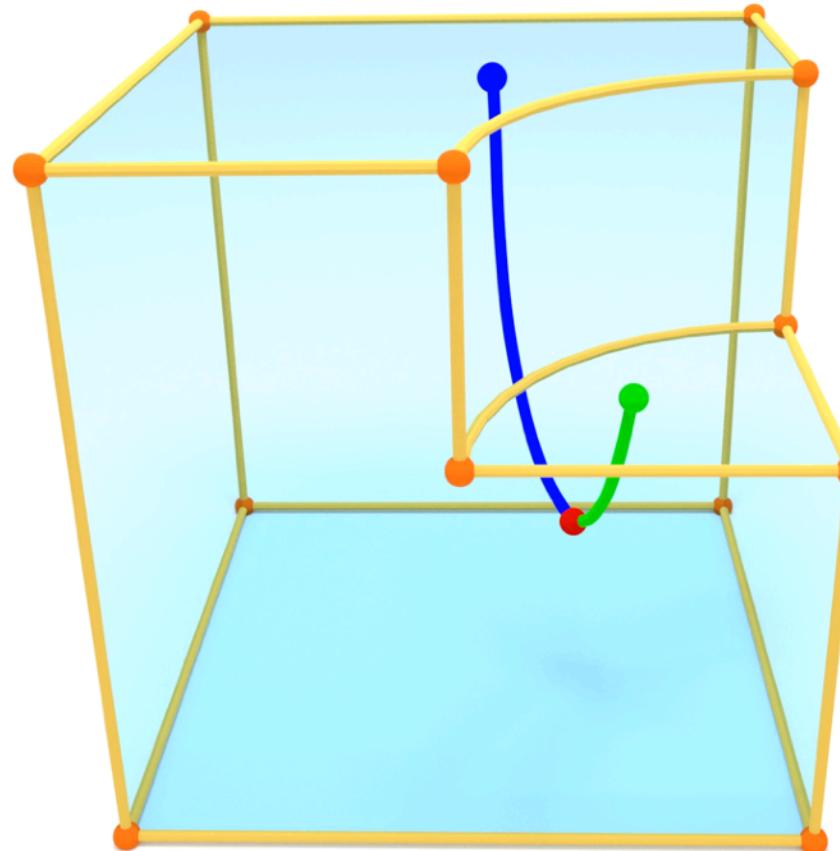
- Smoothest Fields contain non-meshable Zipper Nodes





REPAIRING ZIPPER NODES

- Resolve partial overlap by **unzipping** along common streamline





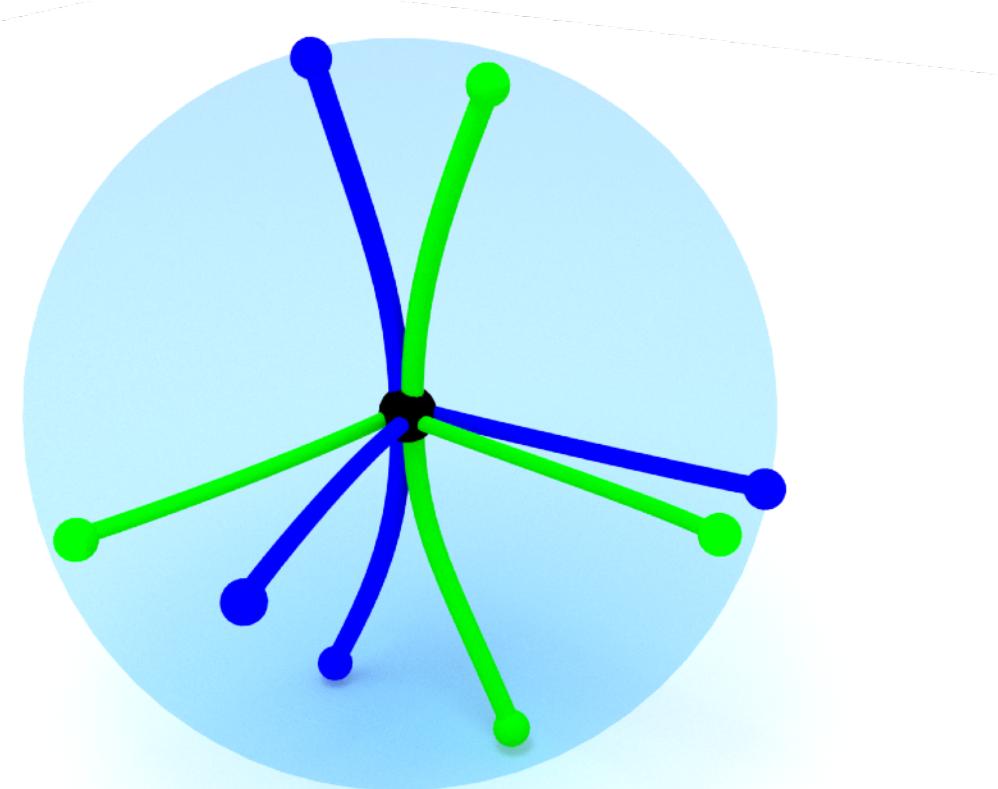
REPAIRING GENERAL NODES



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- **Decomposition Theorem**

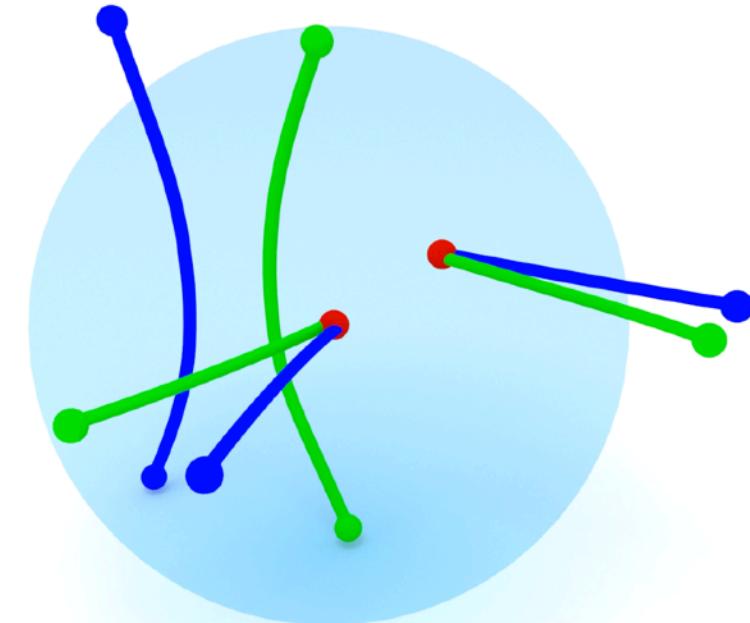
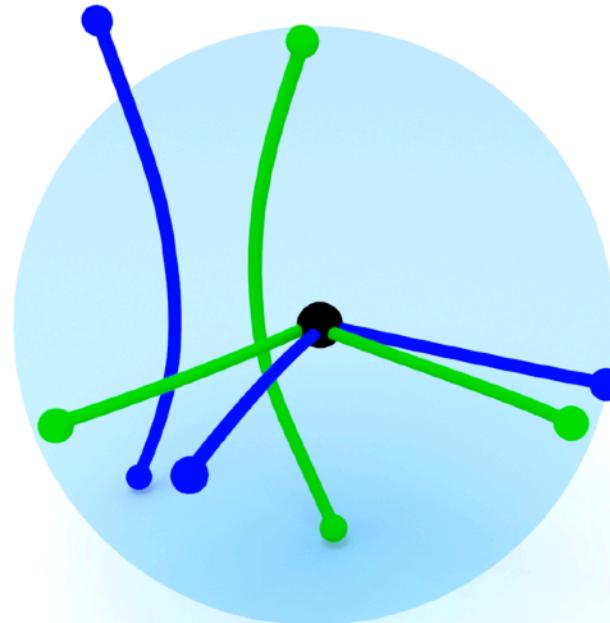
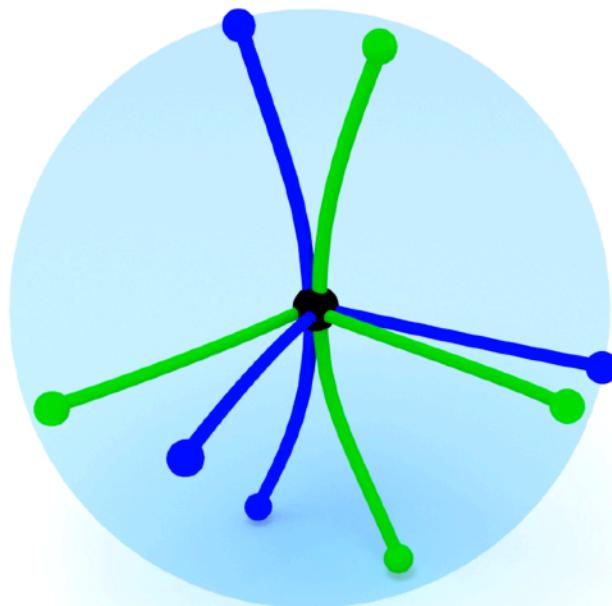
Every singular node can locally be decomposed into isolated singular arcs and zipper nodes.





REPAIRING GENERAL NODES

- Detaching arcs
- Detaching zipper nodes
- Repairing zipper nodes



Algorithm



ACHIEVING LOCAL MESHABILITY



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Algorithm

1. Repair non-meshable **arcs** ← local modifications
2. Decompose non-meshable **nodes** ← local modifications
3. Repair **zipper nodes** ← global modifications

Handling Noisy Singularities



SINGULARITY GRAPH OPTIMIZATION



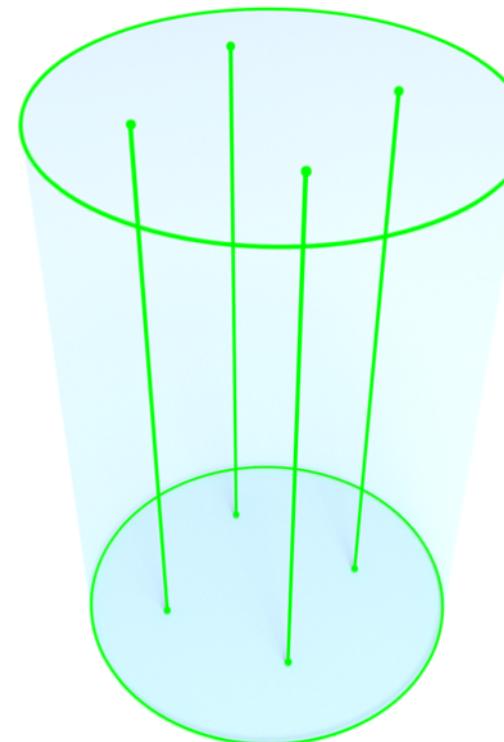
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- **Discrete Singular Arcs**

- are edges of input tetrahedral mesh
- not necessarily align to frame field
- often contain many zipper nodes

- **Strategy**

- delay zipper node repair
- geometrically optimize singular arcs
- co-optimize frame field and tet mesh

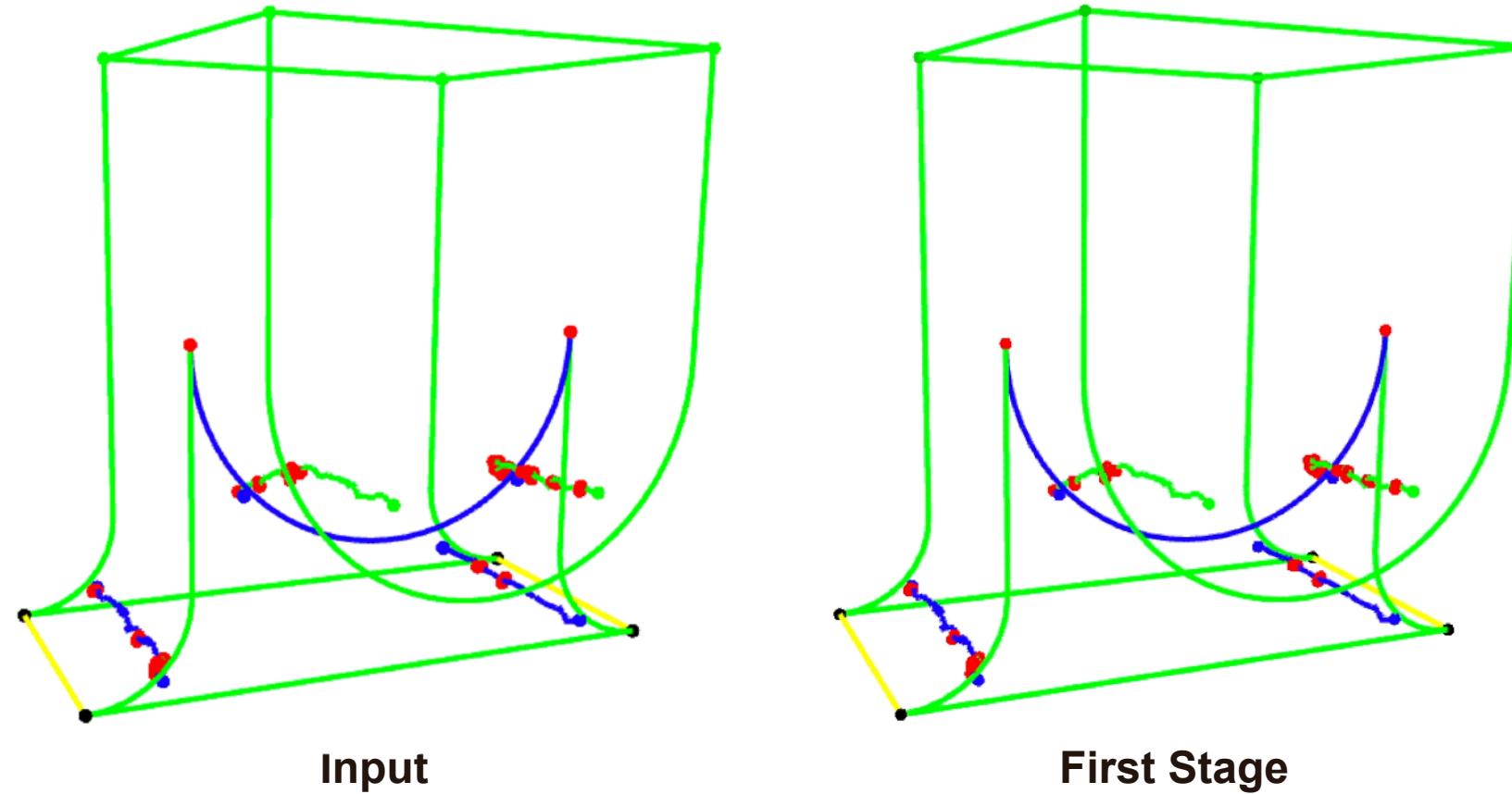




TWO-STAGE ALGORITHM

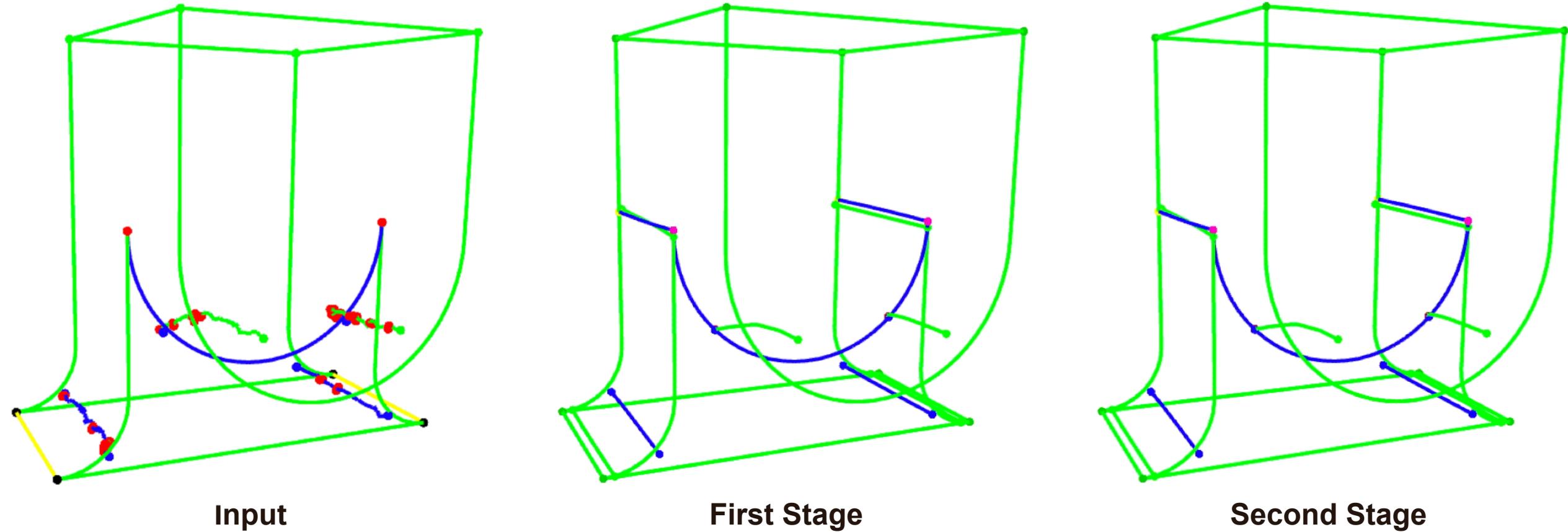


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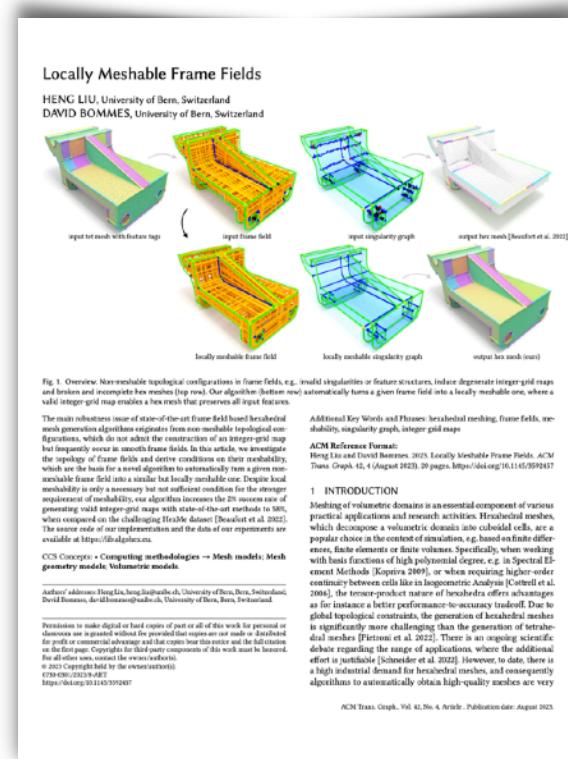
TWO-STAGE ALGORITHM





ADDITIONAL ASPECTS

- Feature Constraints
 - Curves & Surfaces
- Zipper Node Repair
- Discretization on Tetrahedral Mesh
- Singularity Graph Optimization
- ...



Locally Meshable Frame Fields

HENG LIU, University of Bern, Switzerland

DAVID BÖMMES, University of Bern, Switzerland

Fig. 3: Overview of the locally meshable frame fields process. e.g., invalid singularities or feature structures, induce degenerate integer-grid maps and broken and incomplete local meshing (top row). Our algorithm (bottom row) automatically turns a given frame field into a locally meshable one, where a valid integer-grid map makes valid hex meshing possible.

The main robustness issue of state-of-the-art frame field based hexahedral mesh generation algorithms originates from non-meshable topological configurations, which do not admit the construction of an integer-grid map but frequently occur in smooth frame fields. In this article, we investigate how to automatically turn such configurations into locally meshable ones, which are the basis for a novel algorithm to automatically turn a given non-meshable frame field into a similar but locally meshable one. Despite local meshability, our algorithm can handle complex configurations that violate the requirement of meshability; our algorithm increases the 2% success rate of generating valid integer-grid maps with state-of-the-art methods to 58%, when compared on the challenging Helešák dataset [Bouček et al. 2022].

When compared on the challenging Helešák dataset [Bouček et al. 2022], our algorithm significantly outperforms the state-of-the-art methods in terms of success rate and execution time. The execution time of our experiments are available at <https://fb-alphex.ca>.

CCS Concepts: Computing methodologies → Mesh models; Mesh geometry models; Volumetric models.

Authors' addresses: Heng Liu, heng.liu@epfl.ch, University of Bern, Switzerland; David Bömmes, d.bommes@mathweb.msh.inria.fr, University of Bern, Bern, Switzerland.

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ACM Trans. Graph., Vol. 42, No. 4, Article . Publication date: August 2023.

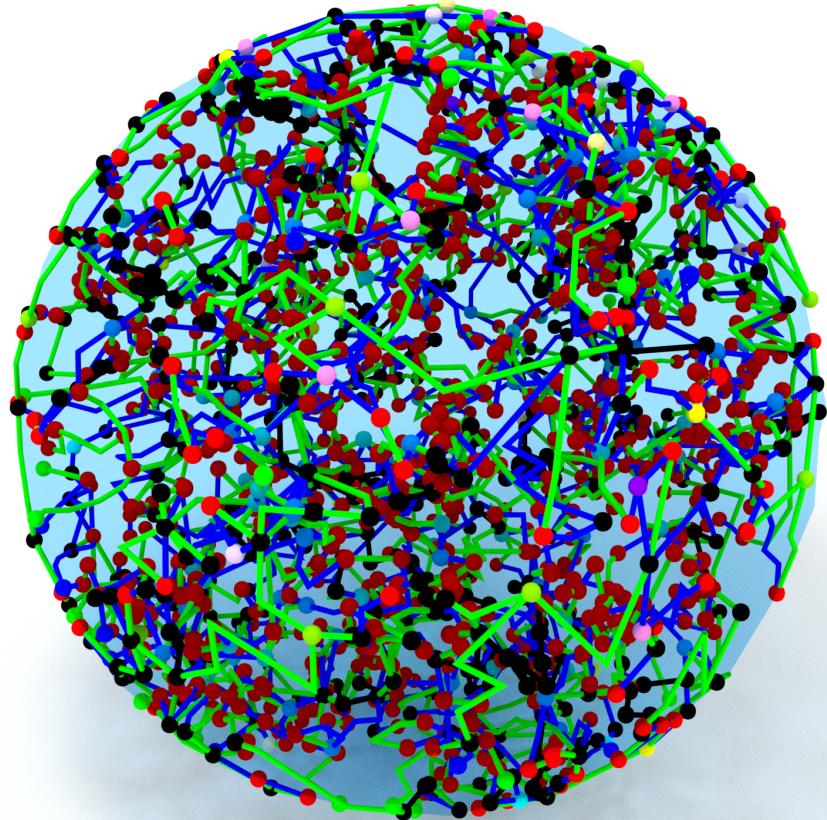
Results



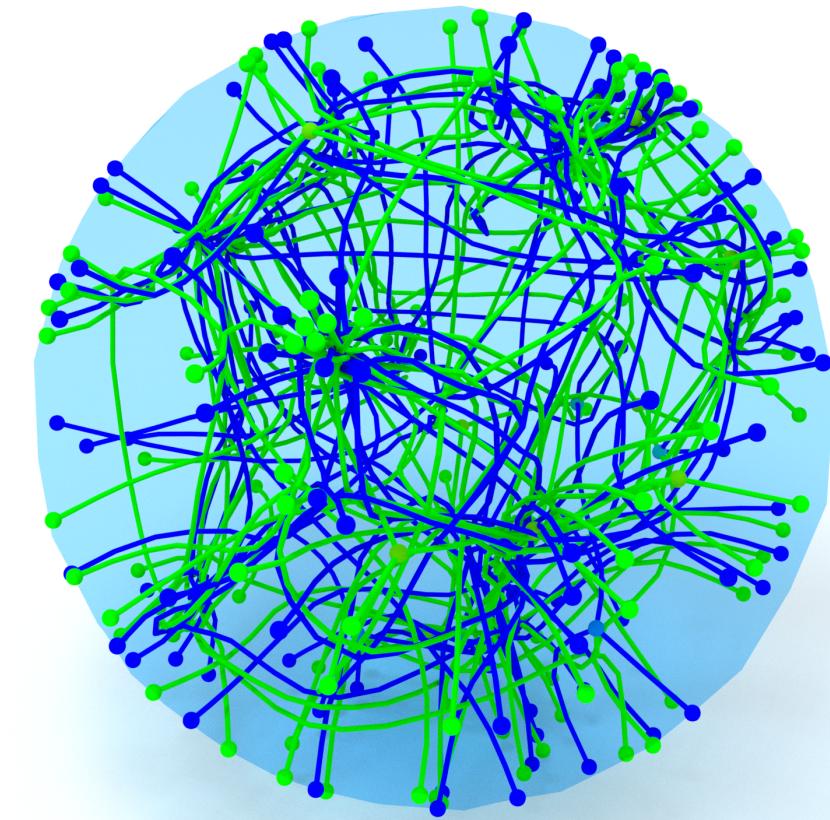
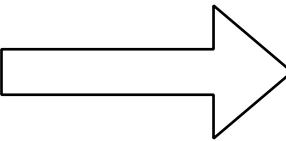
STRESS TEST - RANDOM FRAME FIELD



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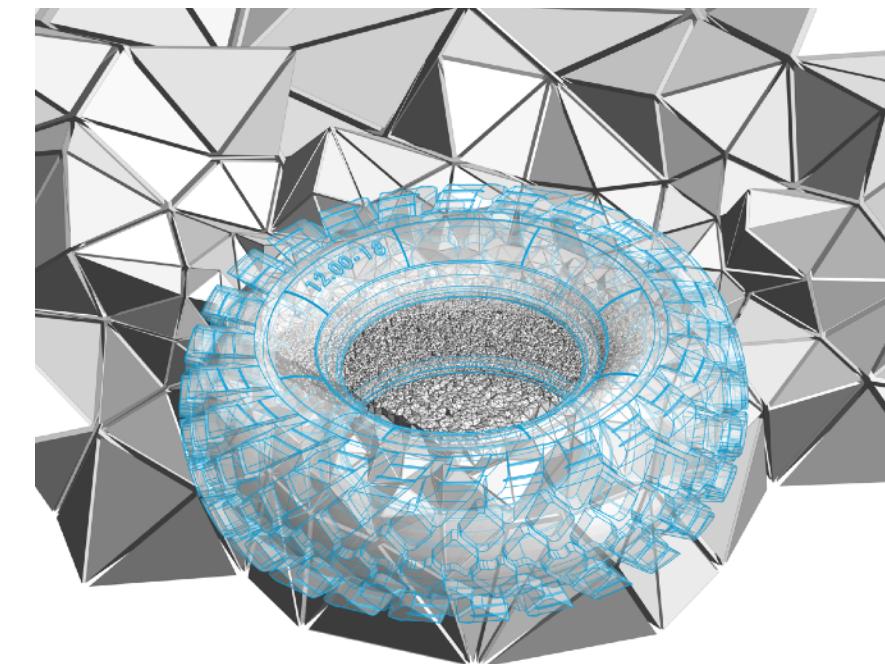
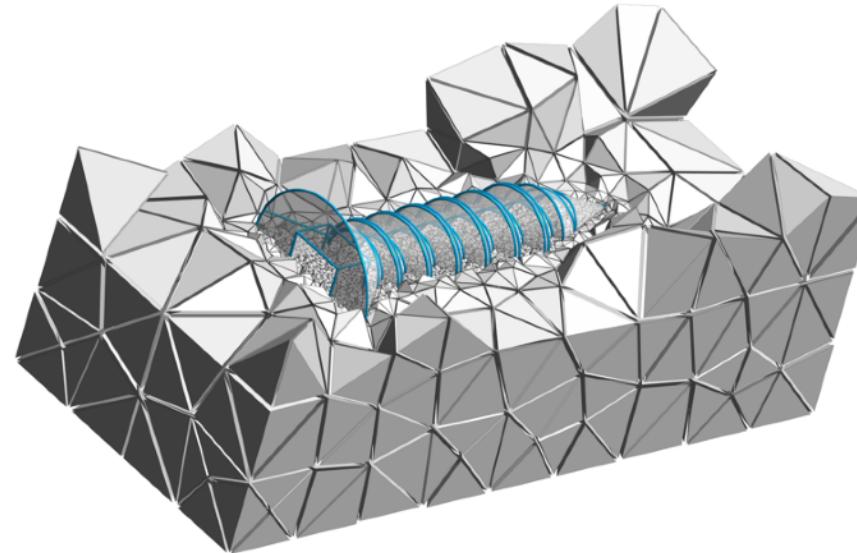
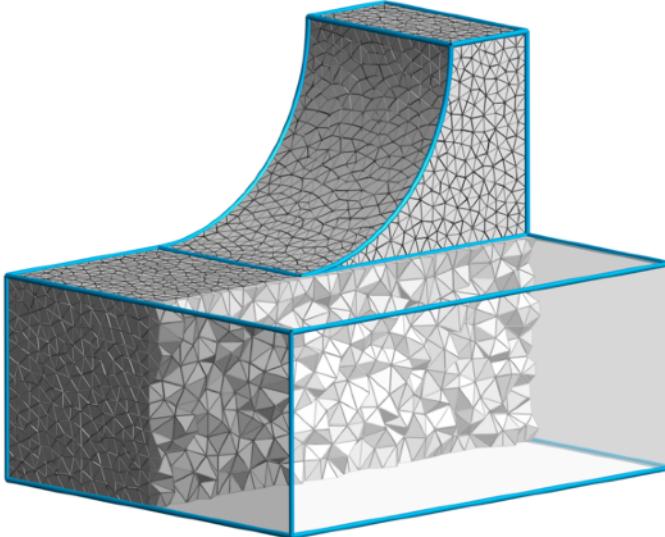
input



locally meshable

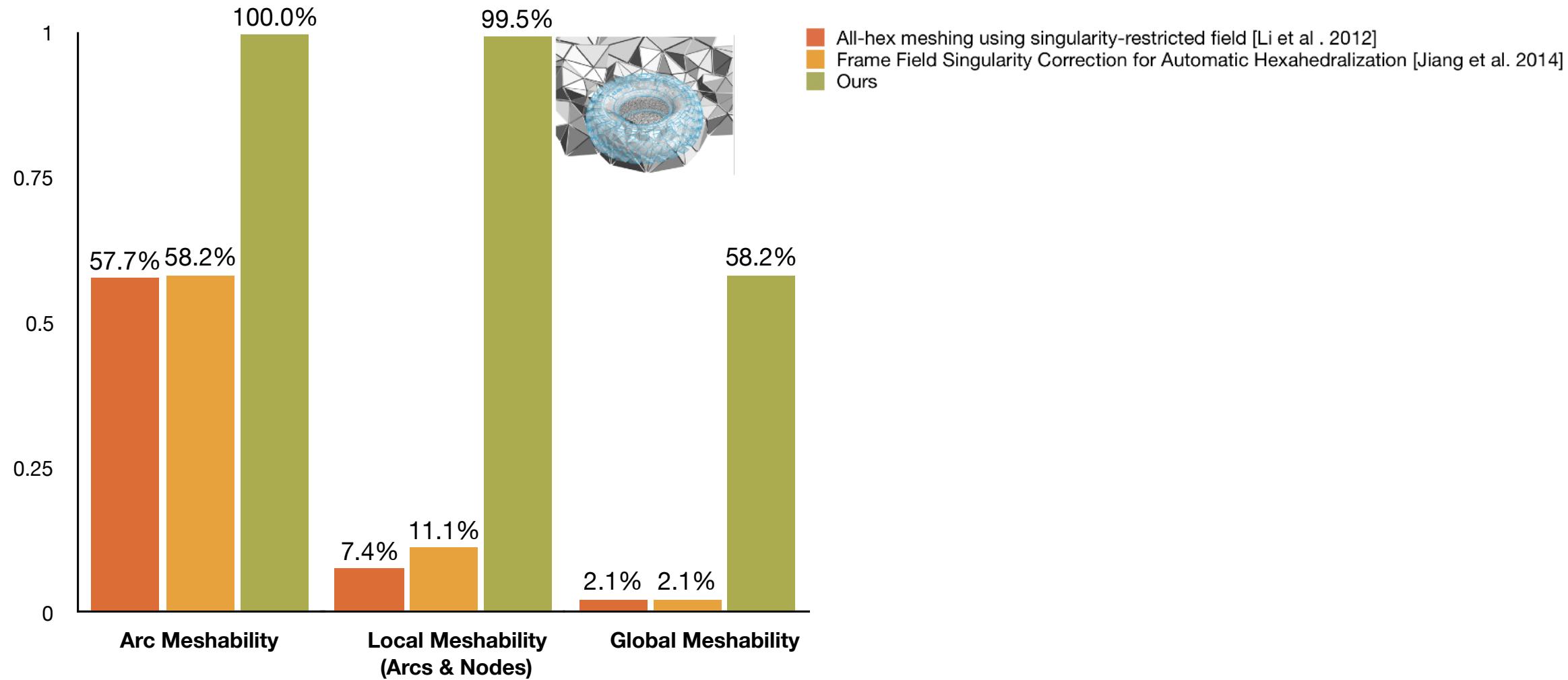
→ EVALUATION

- **HEXME - Dataset** [Beaufort et al. 2022]
 - collection of 189 domains to evaluate and challenge hexmeshers
 - categories: simple, nasty, industrial, including multi-material
 - **goal:** facilitate comparison & guide future research



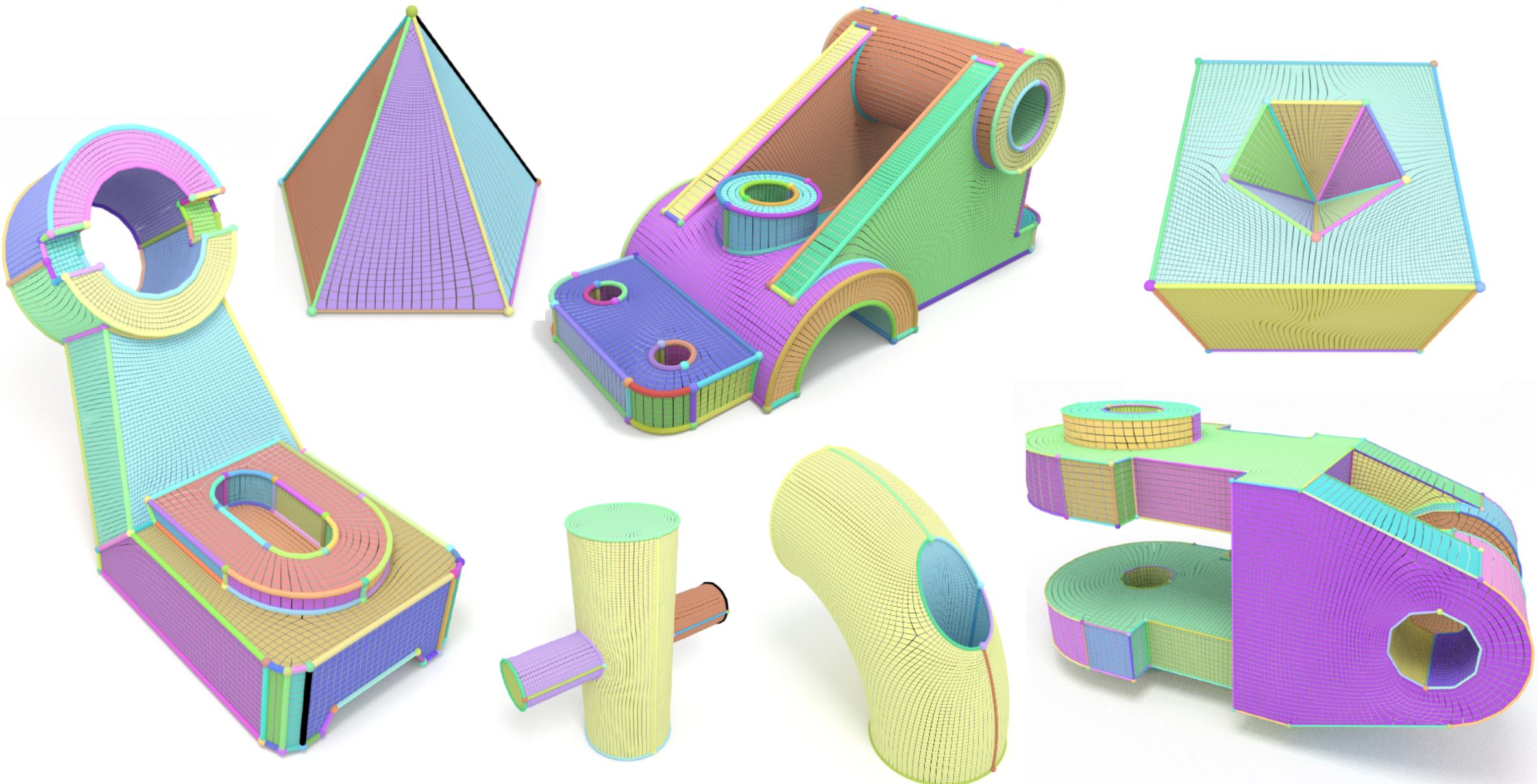


EVALUATION - COMPARISON



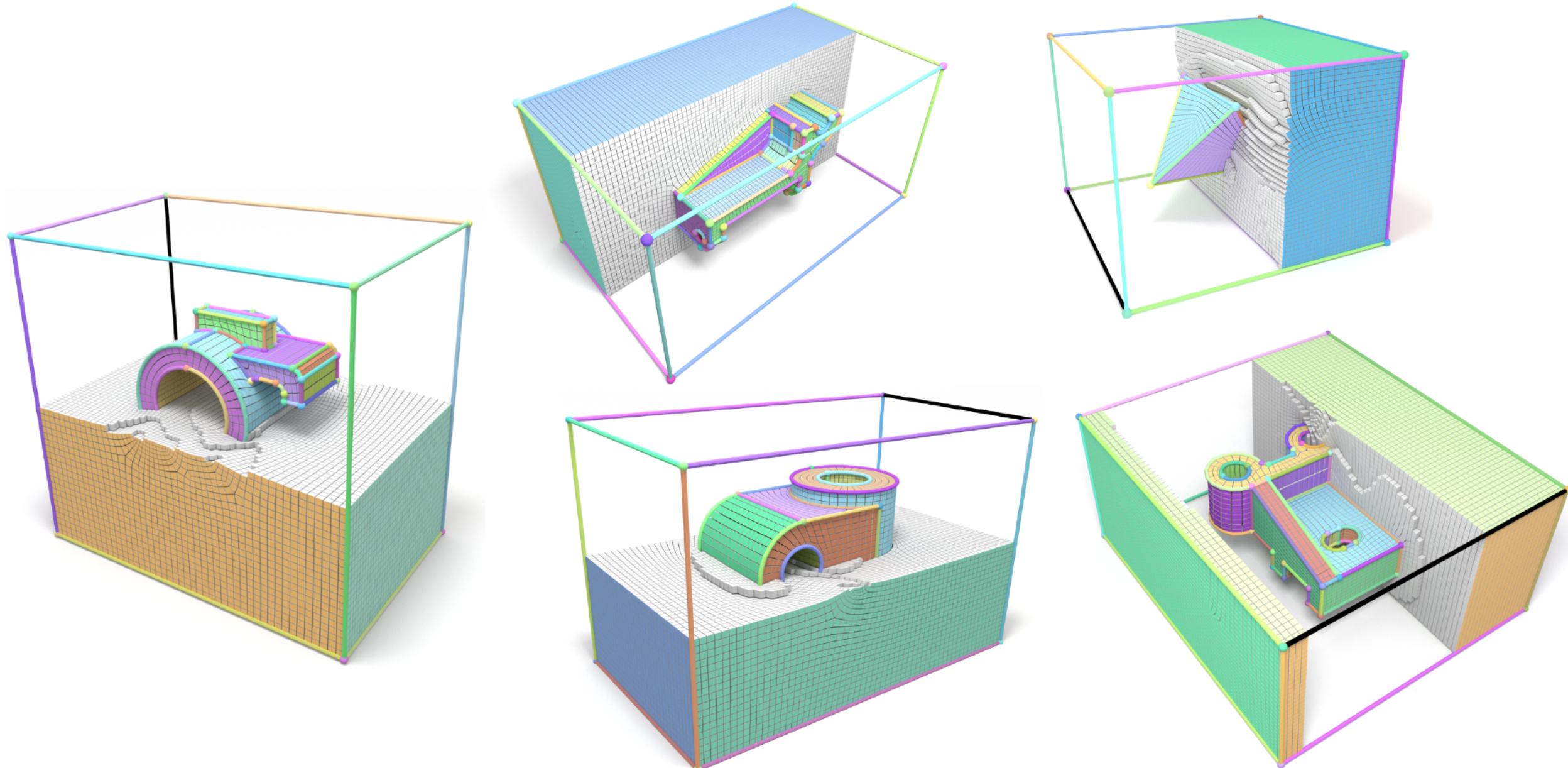


EVALUATION - HEX MESHES



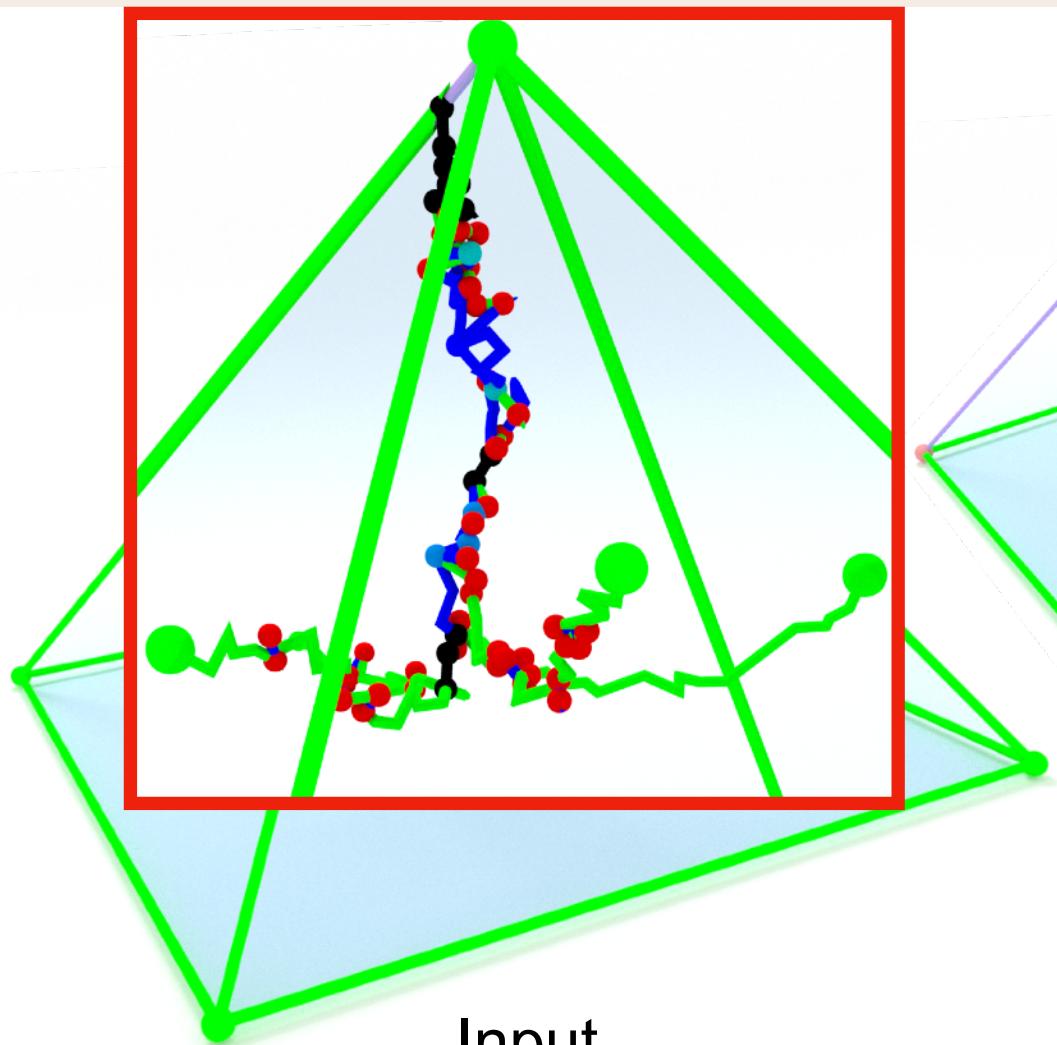


EVALUATION - HEX MESHES

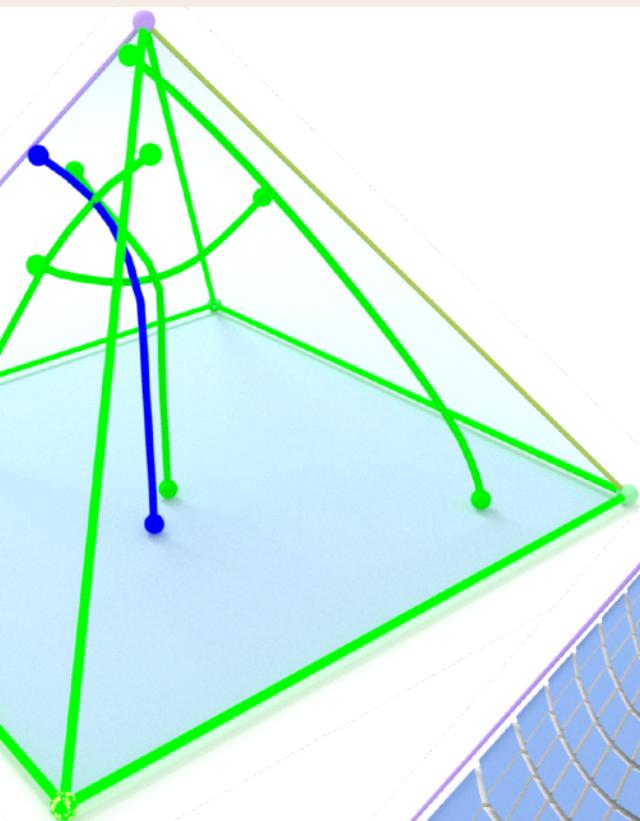




PYRAMID

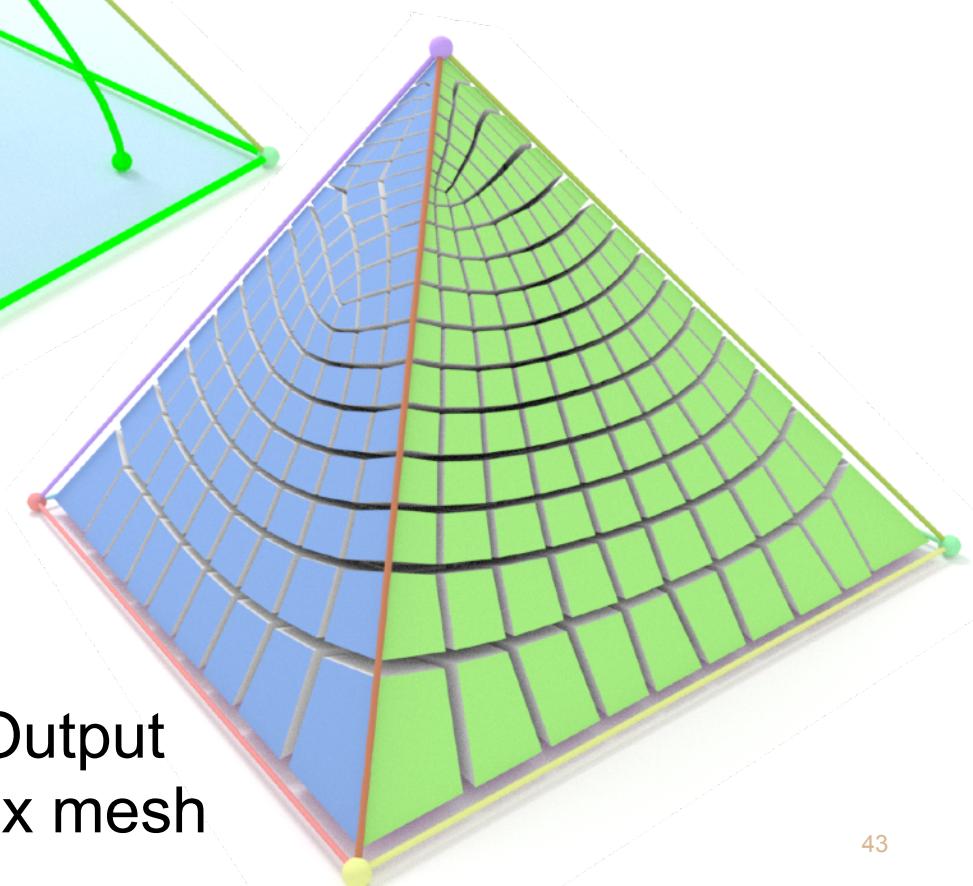


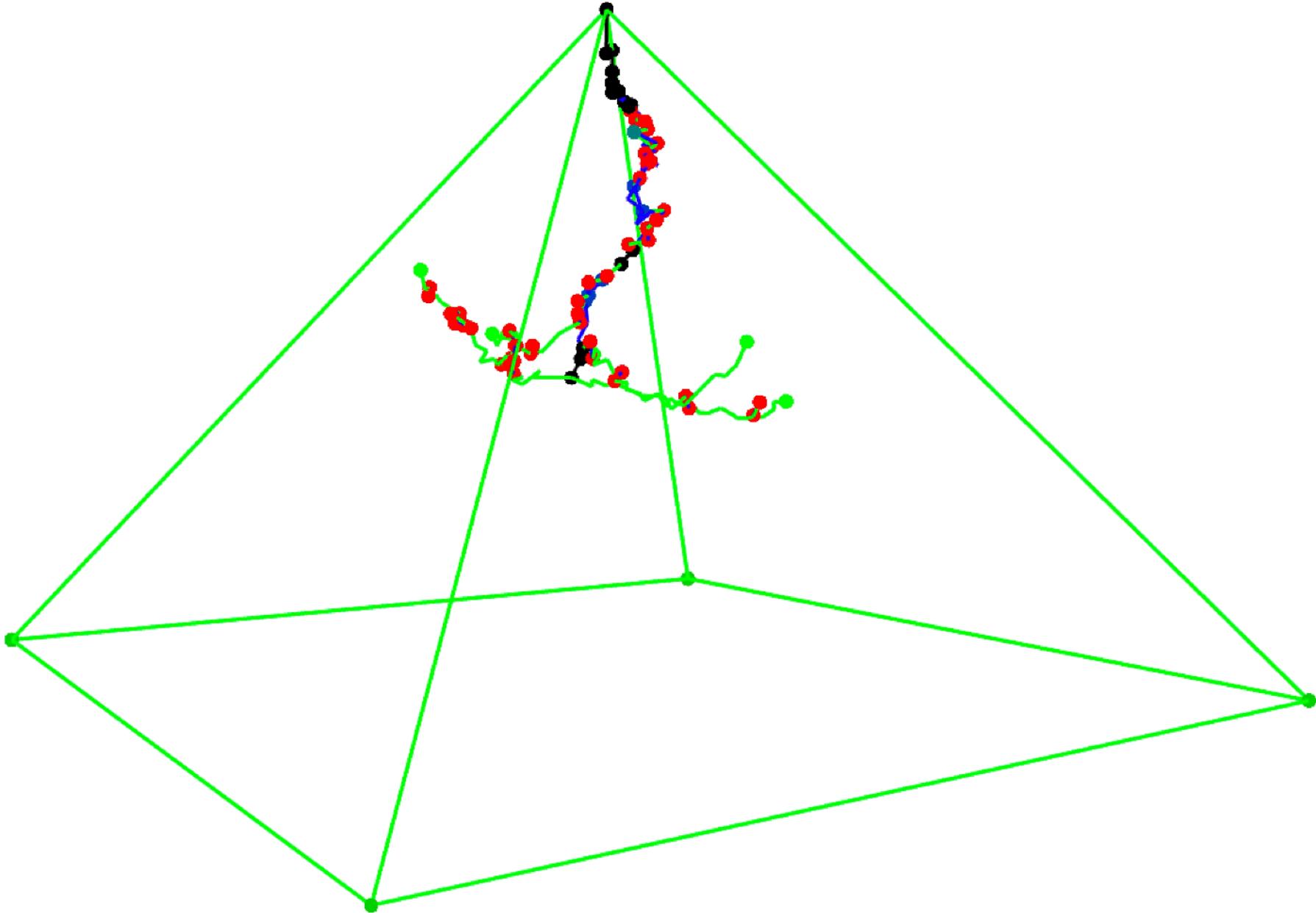
Input



Locally
meshable

Output
hex mesh







MECHANICAL BRACKET - HEXME I29

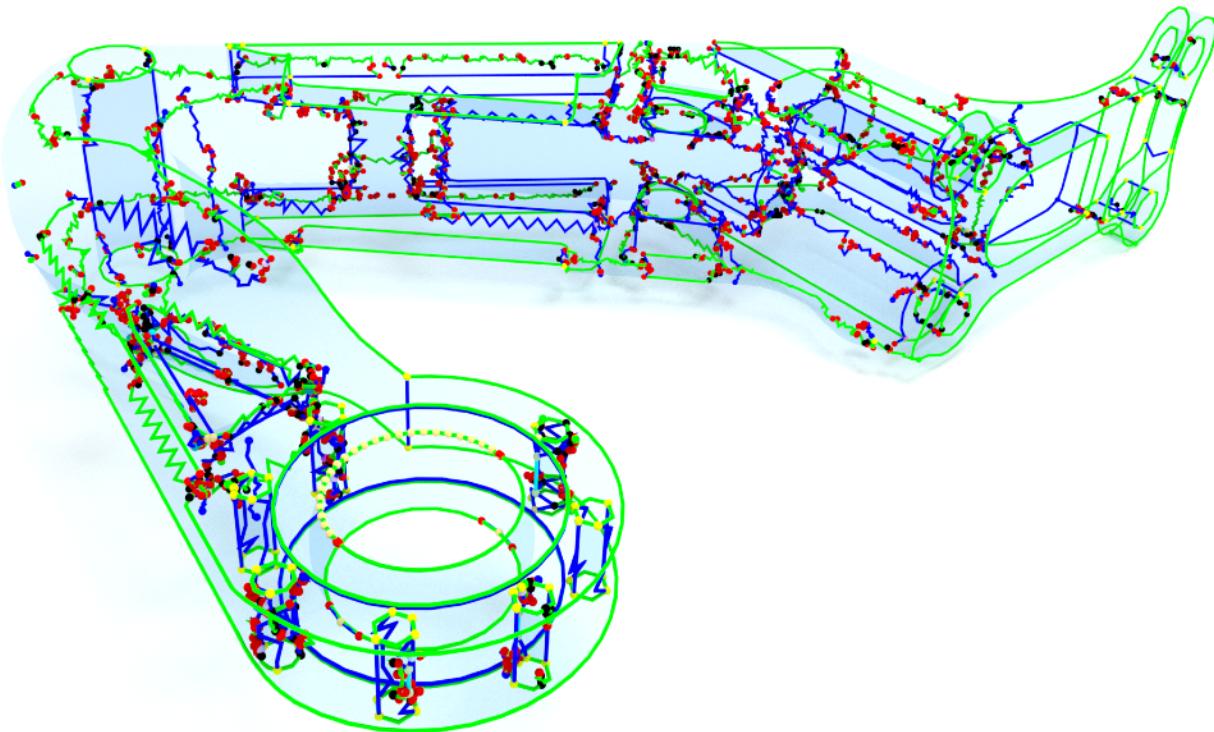


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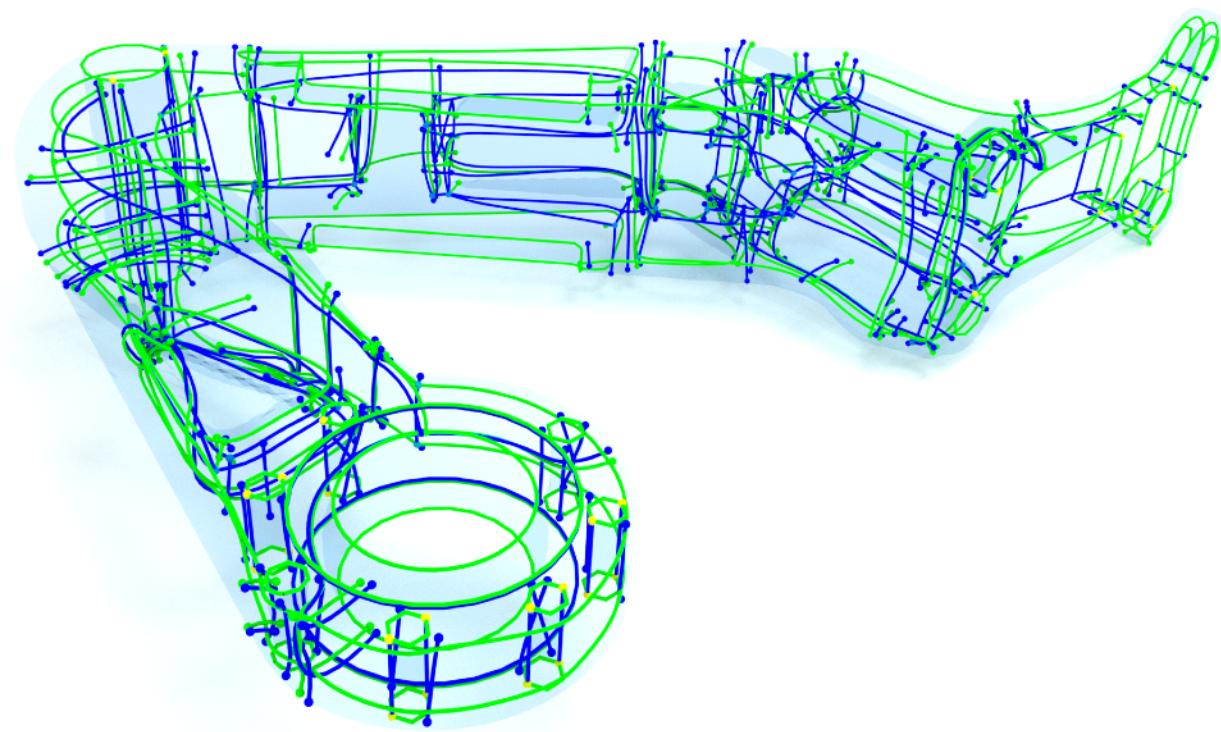




MECHANICAL BRACKET - HEXME I29



input



locally meshable



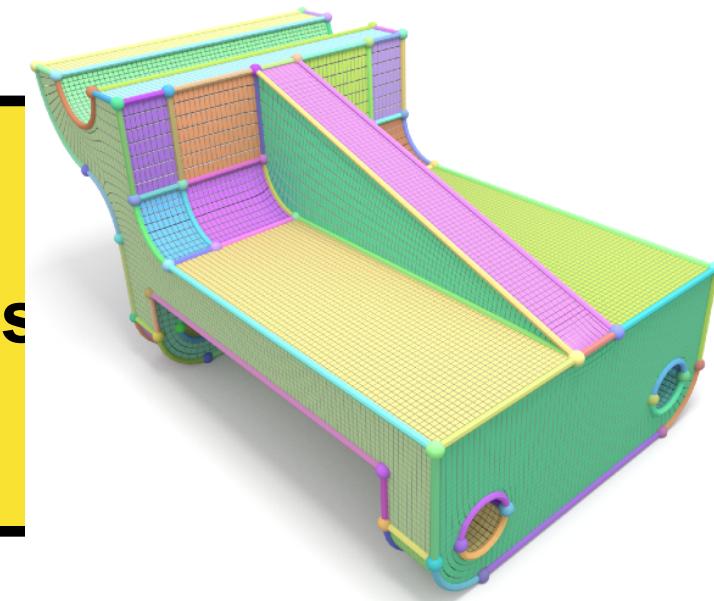
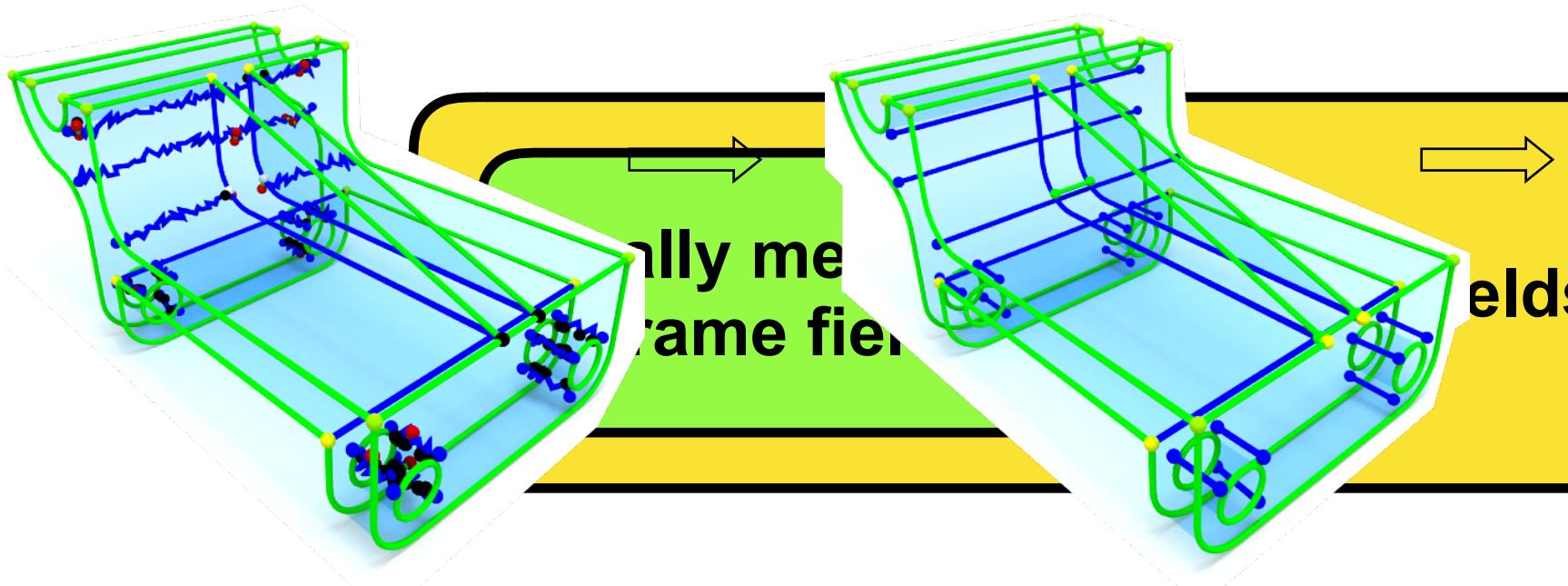
SUMMARY & OUTLOOK



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Contributions

- Characterization of Local Meshability
- Algorithm to ensure Local Meshability





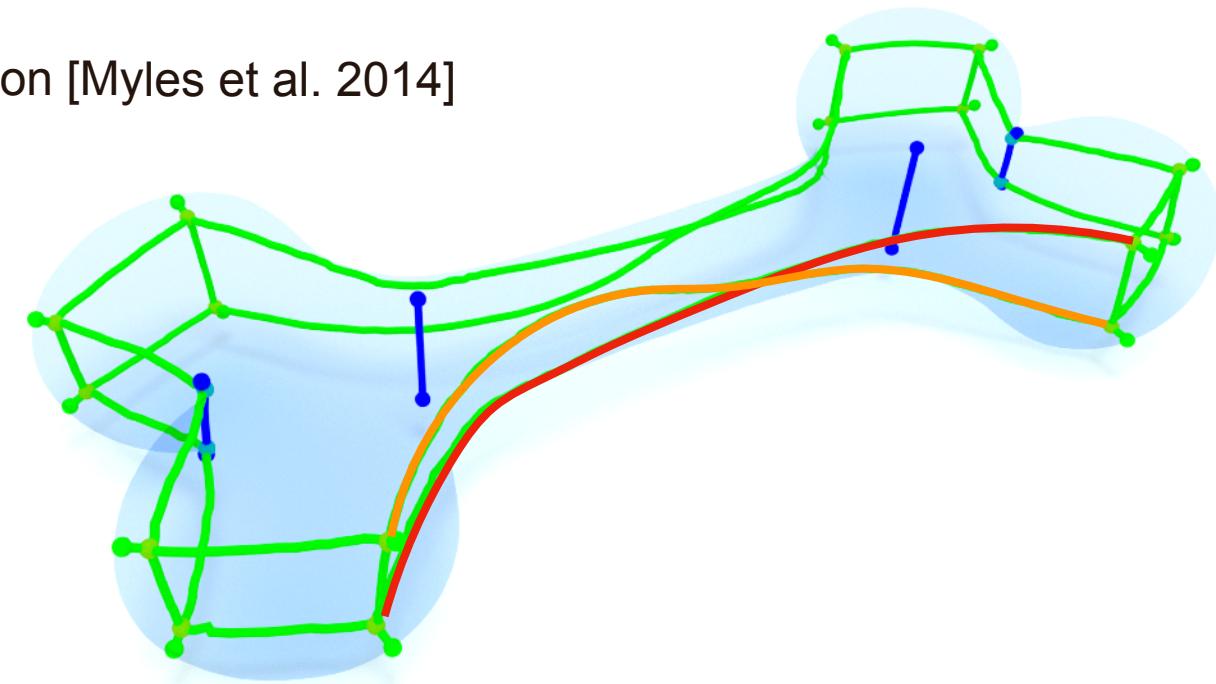
SUMMARY & OUTLOOK



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- **Future Work**

- Global Meshability
 - Solved in 2D: Robust Field-Aligned Parametrization [Myles et al. 2014]
 - Open Problem in 3D





THANK YOU!

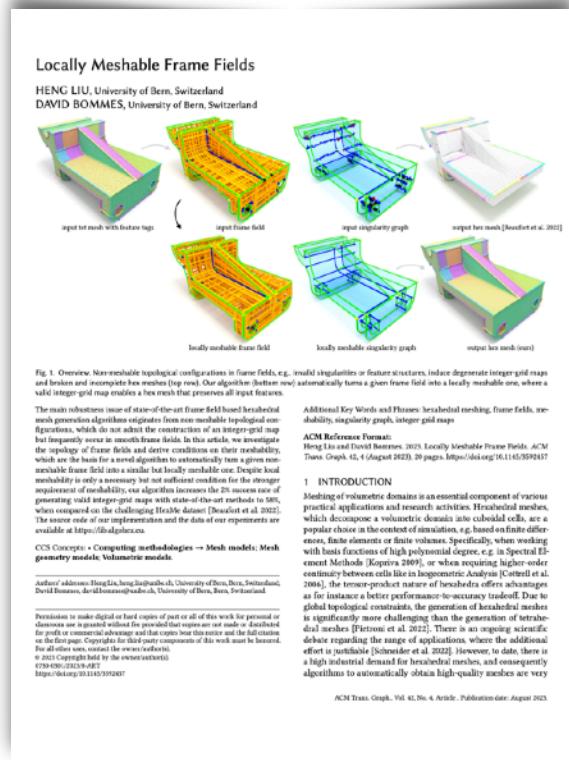


Fig. 1. Overview: Non-meshable topological configurations in frame fields, e.g., invalid singularities or feature structures, induce degenerate integer-grid maps and broken and incomplete hex meshes (top row). Our algorithm (bottom row) automatically turns a given frame field into a locally meshable one, where a valid integer grid map creates a hex mesh that preserves all input features.

The main robustness issue of state-of-the-art frame field based hexahedral mesh generation algorithms originates from non-meshable topological configurations that are not solvable by the existing meshing tools but frequently occur in smooth frame fields. In this article, we investigate the topology of frame fields and derive conditions on their meshability, which we then use to automatically turn them into meshable ones. We turn a non-meshable frame field into a stellar but locally meshable one. Despite local meshability is only a necessary but not sufficient condition for the stronger requirement of meshability, our algorithm increases the 25 success rate of generating hexahedral meshes from 10% to 90% on average and 95% when compared on the challenging HeXele dataset [Beaufort et al. 2022]. The source code of our implementation and the data of our experiments are available at <https://db.algohex.eu>.

CCS Concepts: Computing methodologies → Mesh models; Mesh geometry models; Volume models.

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Code available!